5.7 Gravitational fields, potential and orbits

Name: _____ Class: _____ Date: _____ **Total: 28 marks**

KEY WORDS gravitational field • Gravitational field strength • Gravitational potential

Objective

Develop gravitational fields, potential and orbits from the WPH15 specification and mapped 2025 exam demands. These are original training questions. Use the constants stated in each task.

1 Key knowledge and worked examples

■ Field and potential have different meanings

A **gravitational field** is a region where a mass experiences a force. **Gravitational field strength** $g = F/m$ is force per unit test mass. It is a vector pointing towards an isolated source mass. Newton's law for point masses, or outside a spherical symmetric source, is:

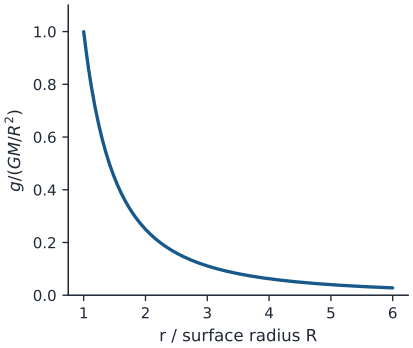
$$F = \frac{GMm}{r^2} \qquad g = \frac{F}{m} = \frac{GM}{r^2}$$

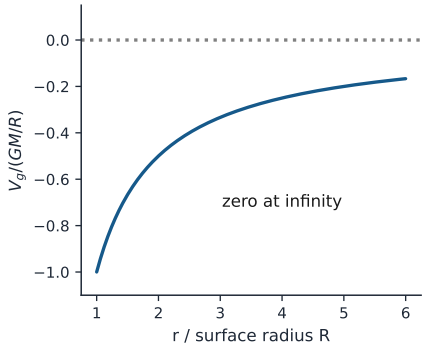
r is centre-to-centre distance, not height above the surface. For near-Earth local motion, constant g can be suitable. Over large radial distances it is not constant.

Gravitational potential is potential energy per unit mass, with zero at infinity:

$$V_{\text{grav}} = -\frac{GM}{r} \qquad E_{\text{grav}} = mV_{\text{grav}} = -\frac{GMm}{r}$$

Potential is a scalar and negative at finite distance. Moving outward makes it less negative, so potential energy increases while attractive force weakens. A potential graph value and its corresponding radius give $M = -Vr/G$.





Gravity and electric fields are both radial for isolated point sources and their force magnitudes obey inverse-square laws. Gravity acts on mass and is attractive in this model. Electric force acts on charge and can attract or repel. Electric field direction follows the force on positive test charge; a negative charge is forced oppositely.

Worked example. A 1000 kg satellite moves from radius $r_1 = 6.4 \times 10^6$ m to $r_2 = 1.28 \times 10^7$ m around Earth, $M = 6.0 \times 10^{24}$ kg. Find its potential-energy change.

$$\Delta E_{\text{grav}} = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\Delta E_{\text{grav}} = (6.67{\times}10^{-11}\ \text{N m}^2\,\text{kg}^{-2})(6.0{\times}10^{24}\ \text{kg})(1000\ \text{kg})\left(\frac{1}{6.4\times10^6\ \text{m}} - \frac{1}{1.28\times10^7\ \text{m}}\right)$$

The positive sign is final minus initial. This is not the total launch energy, since kinetic energy may also change. $mg\Delta h$ with constant surface g is unsuitable over this distance.

■ Derive a circular orbit

The gravitational force supplies the inward resultant:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} = m\omega^2 r$$

$$v = \sqrt{GM/r} \qquad T = 2\pi\sqrt{\frac{r^3}{GM}} \qquad M = \frac{4\pi^2 r^3}{GT^2}$$

Orbiting mass cancels. The period grows with radius. To appear stationary above one point on a rotating planet, a satellite needs a circular equatorial orbit, the same rotational direction and matching angular velocity/period. Matching period alone is insufficient.

Worked example. For $GM = 4.00 \times 10^{14}\ \mathrm{m^3\,s^{-2}}$ and period $T = 8.64 \times 10^{4}\ \mathrm{s}$, find stationary-orbit radius.

$$r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} = \left[\frac{(4.00 \times 10^{14}\ \mathrm{m^3\,s^{-2}})(8.64 \times 10^{4}\ \mathrm{s})^2}{4\pi^2}\right]^{1/3} = 4.23 \times 10^{7}\ \mathrm{m}$$

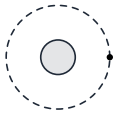
Subtract planet radius only if asked for height above the surface. For a star's surface field, use its radius, not its listed diameter; test a multiple-of-Earth-field claim by calculating the ratio.

2 Practice

2.1 A spherical planet has mass 8.0×10^{23} kg and radius 2.0×10^{6} m. Find surface field strength using $G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$. State its direction. [3]

$M = 8.0 \times 10^{23}$ kg
$R = 2.0 \times 10^{6}$ m

2.2 Gravitational potential at radius 3.0×10^{7} m is $-1.2 \times 10^{7}\ \mathrm{J\,kg^{-1}}$. Find the source mass and explain the negative sign. Use G from 2.1. [4]



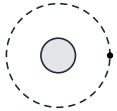
$r = 3.0 \times 10^{7}$ m
$V = -1.2 \times 10^{7}$ J/kg

2.3 Compare gravitational and electric fields of isolated point sources: inverse-square dependence, source property, force directions and potential scalar/vector character. [4]

gravitational field | electric field

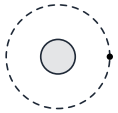
3 Exam-style questions

3.1 A 500 kg probe moves from radius 6.0×10^{6} to 1.8×10^{7} m around a planet with $GM = 3.6 \times 10^{14}\ \mathrm{m^3\,s^{-2}}$. Find the signed change in potential energy. Explain why constant surface g is unsuitable and why your result alone is not total launch energy. [5]



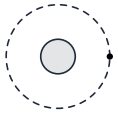
$m = 500$ kg
$r_1 = 6.0 \times 10^{6}$ m
$r_2 = 1.8 \times 10^{7}$ m

3.2 Derive the circular-orbit period equation from Newton's law and centripetal force. A satellite has orbital radius 2.4×10^7 m and period 4.0×10^4 s. Calculate planet mass using G from 2.1. [5]



$r = 2.4 \times 10^7$ m
$T = 4.0 \times 10^4$ s

3.3 A planet rotates every 5.0×10^4 s and has $GM = 8.0 \times 10^{13}$ m^3 s^{-2}, radius 3.0×10^6 m. Find the height of a stationary satellite. State all geometric/directional conditions and explain why an inclined orbit of the same period fails. [7]



$T = 5.0 \times 10^4$ s
$R = 3.0 \times 10^6$ m

Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 Divide Newtonian force by test mass.

$$g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}})(8.0 \times 10^{23}\ \mathrm{kg})}{(2.0 \times 10^{6}\ \mathrm{m})^2} = 13.3\ \mathrm{N\,kg^{-1}}$$

Value; radially inward.

2.2 Potential uses zero at infinity.

$$M = -\frac{Vr}{G} = -\frac{(-1.2 \times 10^{7}\ \mathrm{J\,kg^{-1}})(3.0 \times 10^{7}\ \mathrm{m})}{6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}} = 5.40 \times 10^{24}\ \mathrm{kg}$$

Mass. Positive work is needed to take a unit mass to infinity; its finite-radius potential is below the chosen zero.

2.3 Both force magnitudes vary as inverse square outside a point/spherical source. Gravity acts on mass; electric interaction on charge. Gravity attracts, whereas charges may attract or repel and electric field direction is for a positive test charge. Both field strengths are vectors and both potentials scalars.

3.1 Use final minus initial radial potential energy.

$$\Delta E = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\Delta E = (3.6 \times 10^{14}\ \mathrm{m^3\,s^{-2}})(500\ \mathrm{kg})\left(\frac{1}{6.0 \times 10^{6}\ \mathrm{m}} - \frac{1}{1.8 \times 10^{7}\ \mathrm{m}}\right) = +2.00 \times 10^{10}\ \mathrm{J}$$

Magnitude, positive increase. Field varies substantially over this radial interval. Launch can also change kinetic energy and involve losses.

3.2 Set gravity equal to inward resultant.

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \qquad v = \frac{2\pi r}{T}$$

Eliminate v and cancel orbiting mass.

$$T^2 = \frac{4\pi^2 r^3}{GM} \qquad M = \frac{4\pi^2 r^3}{GT^2}$$

Derivation.

$$M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(2.4 \times 10^{7}\ \mathrm{m})^3}{(6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}})(4.0 \times 10^{4}\ \mathrm{s})^2} = 5.11 \times 10^{24}\ \mathrm{kg}$$

Substitution and result.

3.3 Match rotation period and solve the orbit equation.

$$r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} = \left[\frac{(8.0 \times 10^{13}\ \mathrm{m^3\,s^{-2}})(5.0 \times 10^{4}\ \mathrm{s})^2}{4\pi^2}\right]^{1/3} = 1.72 \times 10^{7}\ \mathrm{m}$$

Radius.

$$h = r - R = (1.7175 \times 10^{7} - 3.0 \times 10^{6})\ \mathrm{m} = 1.42 \times 10^{7}\ \mathrm{m}$$

Height. Orbit must be circular, equatorial and in the planet's rotation direction with matching angular velocity. An inclined orbit changes latitude relative to the surface, so does not remain over one point.