5.6 Forced oscillations, resonance and damping

Name: _____ Class: _____ Date: _____ **Total: 28 marks**

KEY WORDS free oscillation • forced oscillation • Natural frequency • Resonance • Damping • Plastic deformation

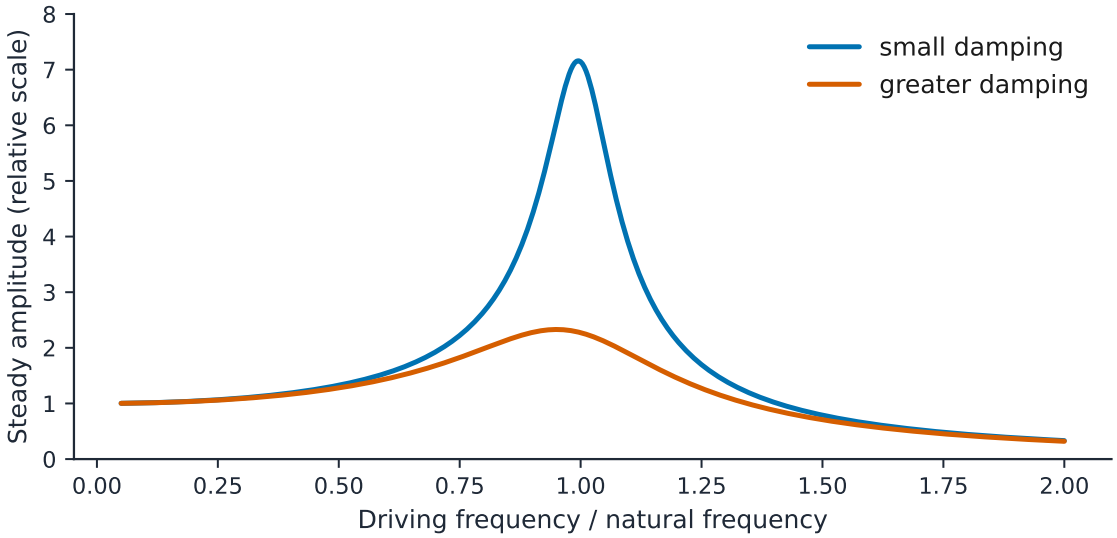
Objective

Develop forced oscillations, resonance and damping from the WPH15 specification and mapped 2025 exam demands. These are original training questions. Use the constants stated in each task.

1 Key knowledge and worked examples

A **free oscillation** follows an initial disturbance without continued periodic driving. A **forced oscillation** is maintained by a periodic driving force; its steady frequency is the driving frequency. **Natural frequency** is the frequency of free oscillation for the system under the stated conditions.

Resonance occurs when driving frequency is at or near natural frequency, giving efficient energy transfer and a large steady amplitude. It does not mean "maximum frequency". For a lightly damped system, the amplitude peak is near the undamped natural frequency. Greater damping lowers and broadens the peak; do not present every damped peak as exactly at the undamped natural frequency.



Damping transfers energy away from oscillation, for example through resistive work that increases the thermal energy of dampers and surroundings. Stronger damping can reduce dangerous bridge/building motion. **Plastic deformation** of a ductile material also absorbs mechanical energy irreversibly; purely elastic deformation returns stored energy and is not the same mechanism.

Worked explanation. People walking supply a periodic driving force to a bridge. If its frequency is near a natural frequency, energy transfers efficiently into the bridge and amplitude increases. Dampers do work against motion, transferring oscillation energy to thermal stores, so the steady amplitude is limited. Link cause and effect in this order rather than listing "resonance, energy, damping".

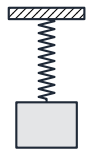
For the same shape and amplitude, adding mass to a light pendulum increases its stored mechanical energy while the small-angle ideal period remains unchanged. Under comparable resistive losses it can lose a smaller fraction of energy per cycle. That contextual damping comparison does not prove that every heavier oscillator always damps more slowly.

2 Practice

2.1 A motor drives a platform at 2.0 Hz, while its undriven small-amplitude natural frequency is 1.6 Hz. State the steady forced frequency and distinguish forced from free oscillation. [3]

drive 2.0 Hz; natural 1.6 Hz

2.2 A damper removes 0.020 J of mechanical energy each cycle from an oscillator initially storing 0.10 J. Calculate the remaining energy after two cycles and the amplitude ratio, assuming spring constant unchanged. [4]



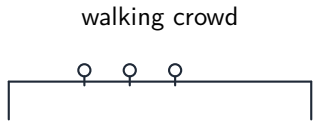
$E_0 = 0.10$ J
loss 0.020 J/cycle

2.3 Two otherwise identical small-angle pendulums have bob masses m and $3m$, equal length and equal angular amplitude. Compare periods and stored mechanical energies. Explain why these facts alone do not fix their damping times. [4]

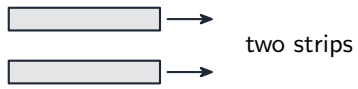
bobs m and $3m$
equal l and angle

3 Exam-style questions

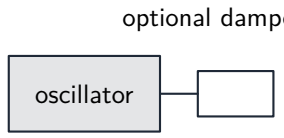
3.1 A bridge has a natural lateral frequency near a crowd's stepping frequency. Explain the sequence from periodic forcing to potentially dangerous motion, then explain how a damper limits it. Distinguish driving frequency from amplitude. [6]



3.2 A protective strip is stretched beyond its elastic limit while a second strip is deformed only elastically. Explain which mechanism can absorb energy irreversibly and why elastic storage alone does not guarantee damping. Distinguish ductile from brittle failure. [5]



3.3 A team compares forced amplitude–frequency curves with and without an added damper. Describe a fair measurement method, predict two changes in the peak, and explain why the frequency of the greatest amplitude need not remain exactly the undamped natural frequency. [6]



Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 The steady forced frequency is 2.0Hz. Forced oscillation is maintained by continued periodic driving; free oscillation follows a disturbance without continued periodic forcing.

2.2 Subtract transferred energy from the initial store.

$$E_2 = E_0 - 2\Delta E = (0.10 - 2 \times 0.020)\ \text{J} = 0.060\ \text{J}$$

Energy. Spring energy is proportional to amplitude squared.

$$\frac{A_2}{A_0} = \sqrt{\frac{E_2}{E_0}} = \sqrt{\frac{0.060\ \text{J}}{0.10\ \text{J}}} = 0.775$$

Ratio. Constant loss per cycle is stated for these two cycles, not a universal damping law.

2.3 Ideal periods are equal because $T = 2\pi\sqrt{l/g}$ does not depend on mass. Equal length and angle give equal vertical rise; $E = mgh$ therefore gives energy ratio 1:3. Damping time also depends on the resistive mechanism and losses per cycle; those have not been specified.

3.1 Walking provides a periodic driving force. Near a natural frequency, energy transfer into the relevant bridge mode is efficient; its amplitude grows. Large amplitude is the danger, not a claim that the bridge frequency grows indefinitely. Dampers do work against motion, transferring mechanical energy to thermal stores and limiting steady amplitude.

3.2 Plastic deformation persists after unloading and transfers mechanical energy irreversibly into internal energy. An ideal elastic strip returns its stored energy, so storage alone is not loss/damping. Ductile material can undergo substantial plastic deformation before fracture; brittle material fractures with little plastic deformation.

3.3 Keep driving-force amplitude and oscillator mass/spring/support unchanged. Sweep driving frequency and measure settled amplitude at each frequency, sampling densely near the peak and repeating. Added damping lowers the peak and broadens it. The amplitude maximum depends on damping as well as restoring/inertial properties, so stronger damping can shift it below the undamped natural frequency; only light damping makes them approximately equal.