5.3 Binding energy, fission and fusion

Name: _____ Class: _____ Date: _____ **Total: 28 marks**

KEY WORDS mass deficit • binding energy • unified atomic mass unit • Nuclear fusion • Nuclear fission

Objective

Develop binding energy, fission and fusion from the WPH15 specification and mapped 2025 exam demands. These are original training questions. Use the constants stated in each task.

1 Key knowledge and worked examples

Mass deficit is an energy difference

A nucleus contains Z protons and $A - Z$ neutrons. Its mass is smaller than the total mass of these free nucleons. This **mass deficit** corresponds to the **binding energy** needed to separate the nucleus completely into free nucleons:

$$\Delta m = Zm_p + (A - Z)m_n - m_{\text{nucleus}} \qquad E_b = \Delta m c_0^2$$

Here c_0 is the speed of light, distinguished from specific heat capacity. The **unified atomic mass unit** is $1\ \text{u} = 1.66 \times 10^{-27}$ kg. Convert all masses to the same unit before subtracting. Nuclear masses and neutral-atom masses are not interchangeable without accounting consistently for electrons. Use the masses the question actually supplies.

$$1\ \text{MeV} = 1.60 \times 10^{-13}\ \text{J} \qquad \text{binding energy per nucleon} = E_b/A$$

Worked example. A nucleus has mass deficit 0.030 u and $A = 4$. Find binding energy per nucleon.

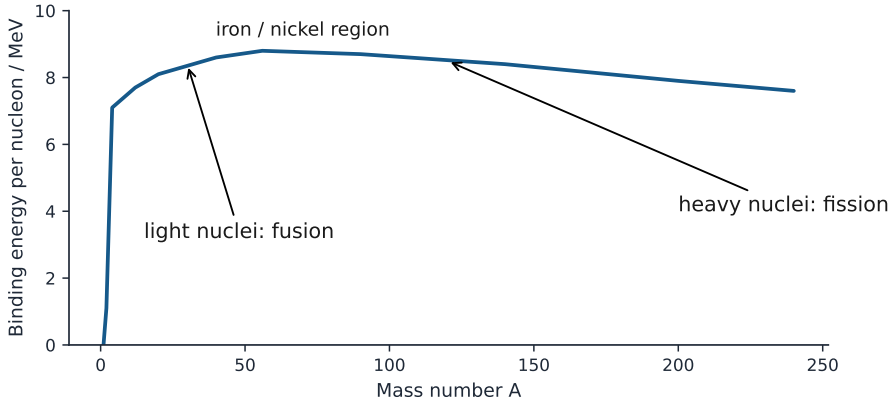
$$\Delta m_{\text{kg}} = \Delta m_{\text{u}}(1.66 \times 10^{-27}\ \text{kg/u}) = 0.030\ \text{u} \times 1.66 \times 10^{-27}\ \text{kg/u} = 4.98 \times 10^{-29}\ \text{kg}$$

$$E_b = \Delta m c_0^2 = 4.98 \times 10^{-29}\ \text{kg} \times (3.00 \times 10^8\ \text{m s}^{-1})^2 = 4.48 \times 10^{-12}\ \text{J}$$

$$\frac{E_b}{A} = \frac{4.48 \times 10^{-12}\ \text{J}}{4 \times 1.60 \times 10^{-13}\ \text{J/MeV}} = 7.00\ \text{MeV per nucleon}$$

Why fusion and fission can both release energy

The binding-energy-per-nucleon curve rises steeply for light nuclei, peaks near iron/nickel, then decreases gradually for heavy nuclei. Greater binding energy per nucleon generally means nucleons are more tightly bound. It is the vertical value, not just mass number, that matters when comparing points.



Nuclear fusion combines light nuclei. **Nuclear fission** splits a heavy nucleus into smaller products, often with neutrons. Energy is released when the products have greater total binding energy and smaller total rest mass than the reactants. For a reaction, calculate the total mass difference, not merely a difference of two per-nucleon values. Total energy is conserved.

Fusion needs very high temperature to give nuclei enough kinetic energy to approach despite electrostatic repulsion, and high density/confinement to make enough close encounters and sustain energy release. Temperature and density have different roles. Merely naming “high pressure” does not explain overcoming repulsion.

2 Practice

2.1 A nucleus has proton number $Z = 8$ and mass number $A = 18$. State proton and neutron numbers. Distinguish binding energy from binding energy per nucleon. [4]

$Z = 8$ $A = 18$

2.2 A nucleus of $A = 12$ has mass deficit 0.098 u. Calculate binding energy in joules and MeV per nucleon. Use 1 u $= 1.66 \times 10^{-27}$ kg, $c_0 = 3.00 \times 10^8$ m s^{-1} and 1 MeV $= 1.60 \times 10^{-13}$ J. [4]

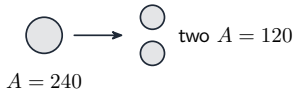
$A = 12$ $\Delta m = 0.098$ u

2.3 Nuclei X and Y have $A_X = 20$, $E_{b,X} = 150$ MeV and $A_Y = 60$, $E_{b,Y} = 510$ MeV. Compare their binding energy per nucleon and identify which has more tightly bound nucleons. [3]

X: 20; 150 MeV Y: 60; 510 MeV

3 Exam-style questions

3.1 A heavy nucleus of mass number 240 with binding energy 7.6 MeV per nucleon splits into two mass-number120 products, each at 8.5 MeV per nucleon. Ignoring other products for this simplified model, find released energy in MeV and joules. Explain its origin without claiming total energy is destroyed. Use 1 MeV $= 1.60 \times 10^{-13}$ J. [5]



3.2 Explain why fusion of some light nuclei and fission of some heavy nuclei both release energy. Explain separately why sustained stellar fusion needs high temperature and sufficiently high density/confinement. [6]

light nuclei heavy nuclei fusion conditions

3.3 A proposed fusion reaction has reactant rest masses totalling 5.030 u and product rest masses totalling 5.010 u. Calculate energy per reaction and the reaction rate for 1.0 MW of released nuclear power. Explain why using a mass difference per nucleon would give the wrong rate. Use the constants from 2.2. [6]

reactants 5.030 u products 5.010 u

Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 Eight protons, ten neutrons. Binding energy is the total energy to separate the nucleus into free nucleons; per-nucleon binding energy divides that total by A.

2.2 Convert mass deficit before applying rest energy.

$$E_b = \Delta m c_0^2 = (0.098\ \text{u})(1.66 \times 10^{-27}\ \text{kg/u})(3.00 \times 10^8\ \text{m s}^{-1})^2 = 1.46 \times 10^{-11}\ \text{J}$$

Total energy. Divide by nucleon count and energy conversion.

$$\frac{E_b}{A} = \frac{1.46412 \times 10^{-11}\ \text{J}}{12(1.60 \times 10^{-13}\ \text{J/MeV})} = 7.63\ \text{MeV per nucleon}$$

Result.

2.3

$$\frac{E_{b,X}}{A_X} = \frac{150\ \text{MeV}}{20} = 7.5\ \text{MeV per nucleon}$$

X.

$$\frac{E_{b,Y}}{A_Y} = \frac{510\ \text{MeV}}{60} = 8.5\ \text{MeV per nucleon}$$

Y. Y has greater binding per nucleon, hence more tightly bound nucleons.

3.1 Compare total binding energies, not just their vertical difference.

$$Q = 2A_p\epsilon_p - A_r\epsilon_r = 2(120)(8.5\ \text{MeV}) - 240(7.6\ \text{MeV}) = 216\ \text{MeV}$$

Energy.

$$Q_J = Q_{MeV}(1.60 \times 10^{-13}\ \text{J/MeV}) = (216\ \text{MeV})(1.60 \times 10^{-13}\ \text{J/MeV}) = 3.46 \times 10^{-11}\ \text{J}$$

Conversion. Products have greater total binding and smaller total rest mass; the decrease becomes kinetic/radiation energy while total energy is conserved.

3.2 The binding-per-nucleon curve peaks near iron/nickel. Appropriate light-nucleus fusion moves towards greater binding per nucleon; heavy-nucleus fission can do the same from the other side. Greater total product binding means lower product rest energy and released energy. High temperature gives nuclei enough kinetic energy for close approach despite electrical repulsion; density/confinement gives sufficient close encounters and retains the reacting material.

3.3 Find total reactant-minus-product mass.

$$\Delta m = m_r - m_p = (5.030 - 5.010)\ \text{u} = 0.020\ \text{u}$$

$$Q = \Delta m c_0^2 = (0.020\ \text{u})(1.66 \times 10^{-27}\ \text{kg/u})(3.00 \times 10^8\ \text{m s}^{-1})^2 = 2.99 \times 10^{-12}\ \text{J}$$

Energy. Power is energy per reaction times reaction rate.

$$\dot{N} = \frac{P}{Q} = \frac{1.0 \times 10^6\ \text{J s}^{-1}}{2.988 \times 10^{-12}\ \text{J}} = 3.35 \times 10^{17}\ \text{s}^{-1}$$

Rate and unit. Energy per complete reaction is needed; a per-nucleon difference omits the participating nucleon count.