5.2 Ideal gases and molecular kinetic energy

Name: _____ Class: _____ Date: _____ **Total: 30 marks**

KEY WORDS Absolute zero • Absolute temperature • ideal gas • mean square speed • root mean square speed

Objective

Develop ideal gases and molecular kinetic energy from the WPH15 specification and mapped 2025 exam demands. These are original training questions. Use the constants stated in each task.

1 Key knowledge and worked examples

Kelvin and the ideal-gas model

Absolute zero is 0 K, about −273 °C. It is the lower limit of thermodynamic temperature. In the classical ideal-gas model, average translational kinetic energy tends to zero as temperature tends to zero. Real gases condense before that extrapolation is reached; do not claim molecules in every real material lose all possible energy.

Absolute temperature is measured in kelvin: $T = \theta_{\circ\mathrm{C}} + 273$ to the precision normally used here. An **ideal gas** consists of particles with negligible volume, no intermolecular forces except during elastic collisions, and random motion. Pressure comes from momentum changes at walls.

$$pV = Nk_BT$$

Here N is the number of molecules, not the number of moles. Use pressure in pascals, volume in cubic metres and temperature in kelvin. For a fixed number of molecules:

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

At fixed volume, pressure is proportional to kelvin temperature. At fixed temperature, pV is constant. Include external pressure if a gauge reports pressure relative to the atmosphere.

Worked example. A rigid 2.0×10^{-3} m^3 vessel contains gas at 1.0×10^5 Pa and 300 K. Find N and pressure after warming to 360 K.

$$N = \frac{pV}{k_BT} = \frac{(1.0 \times 10^5\ \mathrm{Pa})(2.0 \times 10^{-3}\ \mathrm{m^3})}{(1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}})(300\ \mathrm{K})} = 4.83 \times 10^{22}$$

$$p_2 = p_1\frac{T_2}{T_1} = 1.0 \times 10^5\ \mathrm{Pa} \times \frac{360\ \mathrm{K}}{300\ \mathrm{K}} = 1.2 \times 10^5\ \mathrm{Pa}$$

If one molecule has mass m_0, total gas mass is Nm_0. For a sphere, first find $V = 4\pi r^3/3$; radius is half the diameter.

Derive the molecular energy relation

The kinetic-theory pressure relation is $pV = \frac{1}{3}Nm_0\langle c^2\rangle$, where $\langle c^2\rangle$ is **mean square speed** . It follows from elastic momentum changes at a wall and equal mean squared motion in the three perpendicular directions. The **root mean square speed** is $c_{\mathrm{rms}} = \sqrt{\langle c^2\rangle}$; it is not the square of mean speed.

For a cube of side l, one molecule with velocity component c_x changes wall momentum by $2m_0c_x$ at each elastic collision. Its return time to that same wall is $2l/c_x$. Hence its average force is $m_0c_x^2/l$. Sum over molecules and divide by wall area l^2:

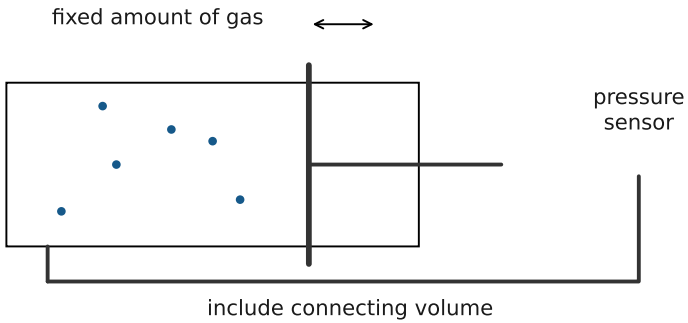
$$p = \frac{Nm_0\langle c_x^2\rangle}{l^3} \qquad V = l^3$$

Random motion is equally distributed among perpendicular directions, so $\langle c^2\rangle = 3\langle c_x^2\rangle$. This gives the pressure relation above. Equate the two expressions for pV, cancel N, then rearrange:

$$\frac{1}{3}Nm_0\langle c^2\rangle = Nk_BT$$

$$\frac{1}{2}m_0\langle c^2\rangle = \frac{3}{2}k_BT \qquad c_{\mathrm{rms}} = \sqrt{\frac{3k_BT}{m_0}}$$

At one temperature, different gas species have equal mean translational kinetic energies, but lighter molecules have greater RMS speeds. If speed doubles at the same temperature, molecular mass is one quarter. Heating increases average molecular kinetic energy; it does not give every molecule the same speed.



■ **CP14: pressure and volume at fixed temperature**

Trap a fixed amount of gas in a syringe or calibrated cylinder connected to a pressure sensor. Check calibration and use absolute pressure. Include connecting-tube dead volume where significant. Change volume gradually, wait after each change for temperature to return to the surroundings, then record several pressure–volume pairs. Do not let gas escape. A plot of p against $1/V$ should be straight through the origin within uncertainty. A p–V curve is not straight. Avoid excessive pressure and secure connections. Rapid compression heats the gas and breaks the intended constant-temperature condition.

2 Practice

2.1 A sealed rigid vessel contains gas at 1.2×10^5 Pa and 27 °C. Find its pressure at 87 °C. Explain why a Celsius ratio fails. [4]

$p_1 = 1.2 \times 10^5$ Pa
27 to 87 °C

2.2 Gas occupies 3.0×10^{-3} m^3 at 2.0×10^5 Pa and 300 K. Find its molecule number using $k_B = 1.38 \times 10^{-23}$ J K^{-1}. Distinguish molecule number from amount in moles. [3]

$p = 2.0 \times 10^5$ Pa
$V = 3.0 \times 10^{-3}$ m^3
$T = 300$ K

2.3 Two ideal gases at the same temperature have molecule masses m and $9m$. Derive their RMS-speed ratio, lighter to heavier, and compare mean translational kinetic energies. [4]

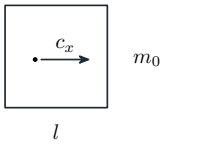
m $9m$ same T

3 Exam-style questions

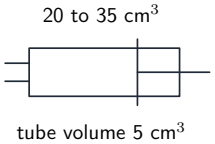
3.1 A spherical gas vessel has internal diameter 0.20 m, pressure 1.5×10^5 Pa and temperature 290 K. Molecule mass is 4.65×10^{-26} kg. Find gas mass and RMS speed. Use $k_B = 1.38 \times 10^{-23}$ J K^{-1}. [6]

$D = 0.20$ m
$p = 1.5 \times 10^5$ Pa
$T = 290$ K

3.2 A particle of mass m_0 moves in a cube of side l. Derive the ideal-gas pressure relation from its wall collisions and isotropic random motion. Combine it with $pV = Nk_BT$ to obtain mean translational kinetic energy. State two ideal-gas assumptions. [7]



3.3 A syringe pressure gauge reads 80 kPa above atmospheric pressure 100 kPa at syringe volume 20 cm^3. Connecting tubing adds 5 cm^3. Find the gauge reading at syringe volume 35 cm^3 after equilibration. Explain two controls and the direction of error if the final gas is still warmer than initially. [6]



Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 Fixed N and V give p/T constant. Convert temperatures to 300K and 360K.

$$p_2 = p_1\frac{T_2}{T_1} = 1.2\times10^5\ \text{Pa}\frac{360\ \text{K}}{300\ \text{K}} = 1.44\times10^5\ \text{Pa}$$

Pressure. Gas-law proportionality uses absolute temperature; Celsius has an offset zero.

2.2 Use the molecule form of the ideal-gas equation.

$$N = \frac{pV}{k_BT} = \frac{(2.0\times10^5\ \text{Pa})(3.0\times10^{-3}\ \text{m}^3)}{(1.38\times10^{-23}\ \text{J K}^{-1})(300\ \text{K})} = 1.45\times10^{23}$$

Molecule count; N counts individual particles, whereas amount in moles is N/N_A.

2.3 From $\frac{1}{2}m_0c_{rms}^2 = \frac{3}{2}k_BT$, RMS speed is inversely proportional to square root of molecule mass at equal T.

$$\frac{c_l}{c_h} = \sqrt{\frac{m_h}{m_l}} = \sqrt{\frac{9m}{m}} = 3$$

Ratio 3:1. Mean translational kinetic energies are equal.

3.1 Radius is half diameter; first calculate volume.

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.10\ \text{m})^3 = 4.19\times10^{-3}\ \text{m}^3$$

$$N = \frac{pV}{k_BT} = \frac{(1.5\times10^5\ \text{Pa})(4.1888\times10^{-3}\ \text{m}^3)}{(1.38\times10^{-23}\ \text{J K}^{-1})(290\ \text{K})} = 1.57\times10^{23}$$

Count.

$$m_g = Nm_0 = (1.5700\times10^{23})(4.65\times10^{-26}\ \text{kg}) = 7.30\times10^{-3}\ \text{kg}$$

Gas mass. Use mean molecular kinetic energy to find RMS speed.

$$c_{rms} = \sqrt{\frac{3k_BT}{m_0}} = \sqrt{\frac{3(1.38\times10^{-23}\ \text{J K}^{-1})(290\ \text{K})}{4.65\times10^{-26}\ \text{kg}}} = 508\ \text{m s}^{-1}$$

Speed and unit.

3.2 Elastic collision gives momentum transfer $2m_0c_x$. The return interval to the same wall is $2l/c_x$; mean force is $m_0c_x^2/l$. Sum N molecules, divide by wall area and use equal squared components in 3directions:

$$pV = Nm_0\langle c_x^2\rangle = \frac{1}{3}Nm_0\langle c^2\rangle$$

Equating with Nk_BT gives $\frac{1}{2}m_0\langle c^2\rangle = \frac{3}{2}k_BT$. Negligible particle volume; negligible forces between particles except elastic collisions.

3.3 Use absolute pressure and total connected volume.

$$p_{1,abs} = p_{1,g} + p_{atm} = (80 + 100)\ \text{kPa} = 180\ \text{kPa}$$

$$p_{2,abs} = p_{1,abs}\frac{V_{1,total}}{V_{2,total}} = 180\ \text{kPa}\frac{(20+5)\ \text{cm}^3}{(35+5)\ \text{cm}^3} = 112.5\ \text{kPa}$$

Absolute result.

$$p_{2,g} = p_{2,abs} - p_{atm} = (112.5 - 100)\ \text{kPa} = 12.5\ \text{kPa}$$

Gauge result. Keep the same sealed amount of gas; change volume slowly and wait for thermal equilibration. If final gas remains warmer, pressure is higher than this isothermal prediction.