

4.6 Nuclear structure, accelerators and tracks

Name: _____ Class: _____ Date: _____ **Total: 33 marks**

KEY WORDS proton number • nucleon number • Thermionic emission
• linear accelerator • cyclotron

Objective

Develop the methods for nuclear structure, accelerators and tracks before the curated WPH14 questions. Use $g = 9.81 \text{ m s}^{-2}$ where needed. These are original training questions, not copied past-paper questions.

1 Key knowledge and worked examples

■ The nucleus and scattering evidence

In ${}_Z^AX$, **proton number** Z counts protons and **nucleon number** A counts protons plus neutrons. The neutron count is $A - Z$. A neutral atom has Z electrons; an ion need not.

The older distributed-positive-charge model could explain small deflections but not the observed rare large-angle alpha scattering. Most alpha particles pass nearly straight through thin foil, showing that most of the atom is empty space. A small fraction experience very large deflections. Positive alpha particles feel strong electrostatic repulsion near a small, dense, positive nucleus containing most atomic mass. Few large-angle events show that the nucleus occupies a small fraction of the atom's volume. Keep each observation linked to the particular conclusion it supports.

■ Make, accelerate and steer a beam

Thermionic emission releases electrons from a heated metal. Heating supplies energy so some electrons can leave the surface. A positive accelerating electrode attracts them through a vacuum. A potential difference changes kinetic energy by the electric work:

$$\Delta E_k = q(V_{\text{initial}} - V_{\text{final}})$$

For speed calculations use the positive energy gain $|q|\Delta V$ only when the particle is accelerated through the field in the appropriate direction. Starting from rest, well

below light speed:

$$\frac{1}{2}mv^2 = |q|\Delta V$$

A **linear accelerator** uses alternating p.d. across gaps between drift tubes. Particles accelerate in gaps and are shielded inside the conducting tubes. The polarity reverses while particles are inside, so the next gap accelerates them again. At constant alternating frequency, increasing speed requires longer tubes to maintain the time between successive gaps. Across n equal accelerating gaps, total kinetic-energy gain is $n|q|V_{\text{gap}}$.

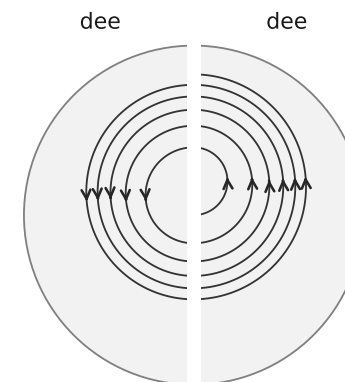
A **cyclotron** uses a magnetic field perpendicular to two hollow dees. Electric field accelerates particles across the gap. Inside a dee, magnetic force curves the path without increasing speed. Polarity reverses during each half-turn, allowing another energy gain at the next crossing. With perpendicular entry:

$$|q|vB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{B|q|} = \frac{p}{B|q|}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{B|q|} \quad f = \frac{B|q|}{2\pi m}$$

In the non-relativistic model, the period is independent of speed. Increasing speed increases radius. Time in one dee is $T/2$, not T . The formula assumes constant field and fixed mass in this model; synchronisation fails as relativistic effects become important.

Cyclotron: radius grows after each gap



alternating gap p.d.; uniform B into page

Worked example. A proton moves perpendicular to $B = 0.40 \text{ T}$ with momentum $2.0 \times 10^{-20} \text{ kg m s}^{-1}$. Use proton mass $1.67 \times 10^{-27} \text{ kg}$.

$$r = \frac{p}{B|q|} = \frac{2.0 \times 10^{-20}}{0.40 \times 1.60 \times 10^{-19}} \text{ m} = 0.313 \text{ m}$$

$$t_{\text{dee}} = \frac{T}{2} = \frac{\pi m}{B|q|} = \frac{\pi \times 1.67 \times 10^{-27}}{0.40 \times 1.60 \times 10^{-19}} \text{ s} = 8.20 \times 10^{-8} \text{ s}$$

When starting instead from a de Broglie wavelength, use $p = h/\lambda$, then $E_k = p^2/(2m)$ and $V = E_k/|q|$. Check that the non-relativistic assumption remains suitable.

■ Read detector evidence carefully

Charged particles ionise matter along their paths, leaving detectable tracks. A neutral particle does not make a direct ionisation track in this model. Its presence can be inferred from missing momentum or from charged products it later produces.

In a known perpendicular magnetic field, curvature direction together with travel direction can identify charge sign. Without field direction, curvature alone cannot give sign. Radius gives momentum through $p = B|q|r$; comparing radii requires knowledge of charge magnitude. A tightening track often shows momentum being lost. Do not infer travel direction from a constant-radius arc alone.

Conserve charge, total energy and vector momentum at an interaction. If visible outgoing momenta do not add to the incoming momentum, a neutral product may carry the missing component. Draw a vector balance rather than guessing from the number of tracks.

High energies probe nucleon structure in two ways. Large momentum gives a short de Broglie wavelength, allowing small structure to be resolved. High collision energy also allows new massive particles to be created. Quarks are not observed as isolated free particles; do not describe a high-energy collision simply as removing a free quark.

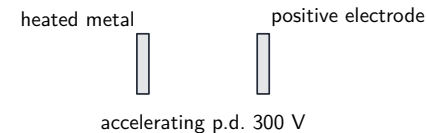
Fast muons can reach the ground because their average lifetime measured in the Earth's frame is increased at speeds close to c . This does not mean they exceed light speed. Unit 4 requires the significance of this lifetime increase, not use of relativistic equations.

2 Practice

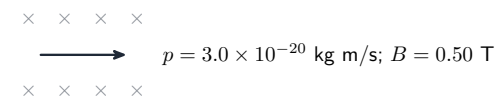
2.1 For a neutral $^{197}_{79}\text{Au}$ atom, state proton, neutron and electron counts. Link two distinct observations from alpha scattering to conclusions about the atom. [5]



2.2 Explain thermionic emission. An electron starts from rest and accelerates through 300 V in vacuum. Calculate its speed using $m_e = 9.11 \times 10^{-31} \text{ kg}$ and $e = 1.60 \times 10^{-19} \text{ C}$, and comment on the non-relativistic assumption. [5]

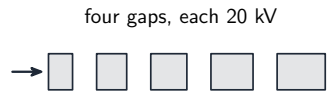


2.3 A proton of momentum $3.0 \times 10^{-20} \text{ kg m s}^{-1}$ enters a perpendicular 0.50 T field. Calculate path radius and time for a half-circle. Use $m_p = 1.67 \times 10^{-27} \text{ kg}$ and $e = 1.60 \times 10^{-19} \text{ C}$. [4]

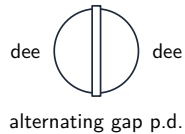


3 Exam-style questions

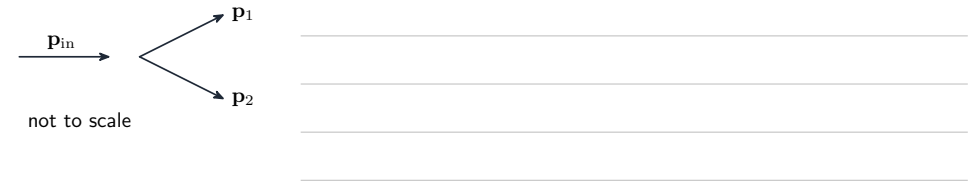
3.1 A proton starts from rest in a linear accelerator and crosses four accelerating gaps, each at 20 kV. Calculate its speed after the fourth gap. Explain why drift tubes become longer at constant supply frequency, why polarity must alternate, and what shielding inside a tube achieves. [6]



3.2 Explain the linked electric and magnetic processes that make a cyclotron accelerate particles repeatedly. Derive its period and explain why increasing radius does not increase the time in a dee in the non-relativistic model. State one limitation near light speed. [6]



3.3 An interaction has incoming momentum $(4.0, 0) \times 10^{-20} \text{ kg m s}^{-1}$. The only visible outgoing charged tracks have momenta $(2.0, 1.0)$ and $(1.0, -0.50)$ in the same units. Find the missing momentum vector and its magnitude. Explain why a neutral product can escape direct tracking, why high-energy collisions help investigate nucleons in two distinct ways, and why fast muons reaching the ground do not imply faster-than-light motion. [7]



Solutions

Original-task marks: M1 = method, A1 = accuracy, B1 = stated reasoning point.

2.1 Protons 79, neutrons $197 - 79 = 118$, electrons 79 for the neutral atom. Most alpha particles pass nearly straight through: most atomic volume is empty. Rare very large deflections require strong repulsion concentrated in a tiny positive nucleus containing most mass, unlike a diffuse-positive-charge model.

2.2 Heating gives some electrons enough energy to leave a metal surface. Electric work becomes kinetic energy.

$$\frac{1}{2}m_e v^2 = eV \quad \Rightarrow \quad v = \sqrt{\frac{2eV}{m_e}}$$

$$v = \sqrt{\frac{2eV}{m_e}} = \sqrt{\frac{2 \times 1.60 \times 10^{-19} \text{ C} \times 300 \text{ V}}{9.11 \times 10^{-31} \text{ kg}}} = 1.03 \times 10^7 \text{ m s}^{-1}$$

Rearrangement and speed. This is about $0.034c$, well below light speed, so the approximation is reasonable.

2.3 Magnetic force supplies centripetal resultant.

$$r = \frac{p}{eB} = \frac{3.0 \times 10^{-20} \text{ kg m s}^{-1}}{1.60 \times 10^{-19} \text{ C} \times 0.50 \text{ T}} = 0.375 \text{ m}$$

Radius. Half the non-relativistic period.

$$t_{1/2} = \frac{\pi m_p}{eB} = \frac{\pi \times 1.67 \times 10^{-27} \text{ kg}}{1.60 \times 10^{-19} \text{ C} \times 0.50 \text{ T}} = 6.56 \times 10^{-8} \text{ s}$$

Half-circle time.

3.1 Add the energy gains of all four gaps.

$$\frac{1}{2}m_p v^2 = 4eV_{\text{gap}} \quad \Rightarrow \quad v = \sqrt{\frac{8eV_{\text{gap}}}{m_p}}$$

$$v = \sqrt{\frac{8eV_{\text{gap}}}{m_p}} = \sqrt{\frac{8 \times 1.60 \times 10^{-19} \text{ C} \times 20 \times 10^3 \text{ V}}{1.67 \times 10^{-27} \text{ kg}}} = 3.92 \times 10^6 \text{ m s}^{-1}$$

Rearrangement and speed. At fixed frequency particles must spend approximately the same half-period inside each tube, so faster motion requires longer tubes. Polarity reverses while the particle is inside so the next gap again accelerates it. Conducting tubes shield the particle from the reversing electric field inside; energy changes mainly in gaps.

3.2 Electric field accelerates across the gap, increasing speed/energy. Perpendicular magnetic force curves motion within dees without doing work. Combining $qvB = mv^2/r$ with $T = 2\pi r/v$ gives:

$$T = \frac{2\pi m}{|q|B}$$

At fixed m, q, B , period is independent of speed; radius grows in proportion to speed and time per dee remains $T/2$. Alternating gap polarity reverses each half-turn to accelerate

again. Relativistic effects alter the required timing near c , so the fixed-frequency non-relativistic synchronisation fails.

3.3 Vector momentum conservation requires subtracting all visible outgoing momenta.

$$\mathbf{p}_{\text{miss}} = \mathbf{p}_{\text{in}} - \mathbf{p}_1 - \mathbf{p}_2 = (1.0, -0.50) \times 10^{-20} \text{ kg m s}^{-1}$$

Components.

$$p_{\text{miss}} = \sqrt{p_x^2 + p_y^2} = \sqrt{1.0^2 + 0.50^2} \times 10^{-20} \text{ kg m s}^{-1} = 1.12 \times 10^{-20} \text{ kg m s}^{-1}$$

Magnitude. Neutral particles do not directly ionise along a track in this detector model; later charged products may reveal them. High momentum gives shorter de Broglie wavelength and finer structural resolution. High collision energy can also create new massive particles. Fast muons have a longer average lifetime in the Earth's frame at speeds close to, but below, c .