

4.3 Electric field and potential

Name: _____ Class: _____ Date: _____ **Total: 28 marks**

KEY WORDS electric field • Electric field strength • Electric potential
• equipotential

Objective

Develop the methods for electric field and potential before the curated WPH14 questions. Use $g = 9.81 \text{ m s}^{-2}$ where needed. These are original training questions, not copied past-paper questions.

1 Key knowledge and worked examples

■ Force and potential describe different things

An **electric field** is a region where a charged particle experiences a force.
Electric field strength is force per unit positive test charge:

$$\mathbf{E} = \frac{\mathbf{F}}{q} \quad \mathbf{F} = q\mathbf{E}$$

Its units are N C^{-1} or V m^{-1} . A positive charge feels force along the field; a negative charge feels force opposite it.

Electric potential is potential energy per unit charge relative to a chosen reference. For a charge q moved through a potential change:

$$\Delta U = q\Delta V \quad W_{\text{field}} = -q\Delta V$$

Potential is a scalar; field is a vector. A point with zero potential need not have zero field. Adding signed potentials is different from adding field vectors.

■ Radial and uniform fields

For point charges, or outside an isolated charged conducting sphere, with $k = 1/(4\pi\epsilon_0)$:

$$|F| = \frac{k|Q_1Q_2|}{r^2} \quad |E| = \frac{k|Q|}{r^2} \quad V = \frac{kQ}{r}$$

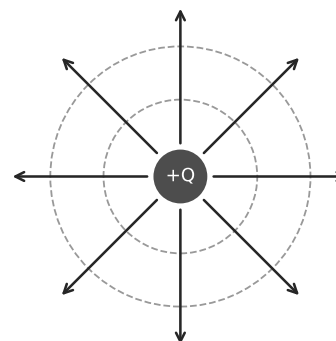
Here r is separation of charge centres; sphere distances are measured from its centre. Like charges repel, unlike charges attract. Potential takes the sign of Q when zero is at infinity.

Between large parallel plates, away from edges, the field is approximately uniform:

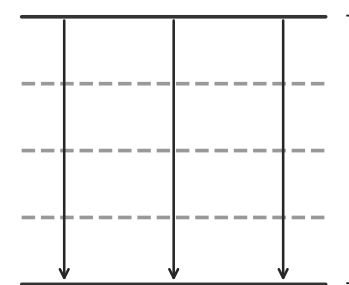
$$|E| = \frac{|\Delta V|}{d}$$

The field points from higher to lower potential. Plate separation d is perpendicular to the plates. An **equipotential** has the same potential everywhere. Field lines cross equipotentials at right angles. Moving along one involves no change in electric potential energy.

Radial field and equipotentials



Uniform field between plates



The radial field is the negative potential gradient: $E_r = -dV/dr$. For a positive source, the signed area under its E - r curve from r to infinity equals $V(r)$. The area from zero to r is not that potential. In a uniform field, closer equally spaced potential levels mean stronger field.

Worked example. A sphere of radius 0.20 m has potential 9.0 kV. Find charge and field at 0.30 m from its centre. Use $k = 8.99 \times 10^9 \text{ N m}^2\text{C}^{-2}$.

$$Q = \frac{Vr}{k} = \frac{9.0 \times 10^3 \text{ V} \times 0.20 \text{ m}}{8.99 \times 10^9 \text{ N m}^2\text{C}^{-2}} = 2.00 \times 10^{-7} \text{ C}$$

$$E = \frac{kQ}{r^2} = \frac{8.99 \times 10^9 \times 2.00 \times 10^{-7}}{0.30^2} \text{ N C}^{-1} = 2.00 \times 10^4 \text{ N C}^{-1}$$

Use the surface radius to find charge, then the new centre-distance to find field. Do not reuse the surface distance for the second step.

■ Compare electrical and gravitational forces

A stationary charged oil drop can satisfy $|q|E = mg$. Use $m = \rho V$ if density and volume are given. Divide the charge magnitude by $e = 1.60 \times 10^{-19} \text{ C}$ to test whether it is close to an integer multiple. Charge sign follows the direction of force required, not its magnitude alone.

To lift a spherical grain, compare qE with $\rho(4\pi r^3/3)g$. Radius is half the listed diameter. A calculated maximum diameter is a threshold: choose the largest listed diameter below it. On an inclined panel, the normal component of weight is $mg \cos \theta$. The electric force perpendicular to the panel need overcome that component to detach a grain; gravity also has a downslope component.

2 Practice

2.1 Parallel plates separated by 0.020 m have p.d. 600 V. Calculate field strength and force magnitude on an electron. State its force direction relative to the field. Use $e = 1.60 \times 10^{-19} \text{ C}$. [4]

○ e^- 600 V; 0.020 m _____

2.2 An isolated conducting sphere has radius 0.10 m and potential +4.5 kV. Find its charge and field strength at 0.30 m from its centre. Use $k = 8.99 \times 10^9 \text{ N m}^2\text{C}^{-2}$. [4]

+4.5 kV
 0.30 m
 r = 0.10 m

2.3 A charge +2.0 nC moves from potential 120 V to 50 V. Find the potential-energy change and work done by the field. Explain why the sign matters. [4]

120 V 50 V
 (+q) →
 q = 2.0 nC

3 Exam-style questions

3.1 Explain how to obtain the potential at distance r from a positive point charge from its $E-r$ graph. Explain the relationship between field direction and equipotentials, and why zero potential at a point does not necessarily mean zero field. [4]

$E-r$ potential equipotentials _____

3.2 A stationary oil drop of mass $6.52 \times 10^{-15} \text{ kg}$ lies between horizontal plates where the field points downwards with magnitude $2.00 \times 10^5 \text{ N C}^{-1}$. Determine the charge, including sign, and the number of excess or missing electrons. State the force-balance assumption. [5]

○ oil drop $E \downarrow = 2.00 \times 10^5 \text{ N/C}$ _____

[7]

Solutions

Original-task marks: M1 = method, A1 = accuracy, B1 = stated reasoning point.

2.1 Uniform field is p.d. per perpendicular separation.

$$E = \frac{V}{d} = \frac{600 \text{ V}}{0.020 \text{ m}} = 3.00 \times 10^4 \text{ V m}^{-1}$$

Field strength.

$$|F| = eE = 1.60 \times 10^{-19} \text{ C} \times 3.00 \times 10^4 \text{ N C}^{-1} = 4.80 \times 10^{-15} \text{ N}$$

Force magnitude; opposite field for the electron.

2.2 Surface potential uses sphere radius.

$$Q = \frac{Vr}{k} = \frac{4.5 \times 10^3 \text{ V} \times 0.10 \text{ m}}{8.99 \times 10^9 \text{ N m}^2\text{C}^{-2}} = 5.01 \times 10^{-8} \text{ C}$$

Positive charge. Use the new centre-distance in the inverse square law.

$$E = \frac{kQ}{r^2} = \frac{8.99 \times 10^9 \text{ N m}^2\text{C}^{-2} \times 5.01 \times 10^{-8} \text{ C}}{(0.30 \text{ m})^2} = 5.00 \times 10^3 \text{ N C}^{-1}$$

Field strength outward.

2.3 Use final minus initial potential.

$$\Delta U = q(V_f - V_i) = 2.0 \times 10^{-9} \text{ C} \times (50 - 120) \text{ V} = -1.40 \times 10^{-7} \text{ J}$$

Energy change.

$$W_{\text{field}} = -\Delta U = -(-1.40 \times 10^{-7} \text{ J}) = +1.40 \times 10^{-7} \text{ J}$$

Work. Potential energy falls as the field does positive work; losing the sign loses that distinction.

3.1 With zero potential at infinity, integrate the signed radial field from r to infinity; this gives potential, not the area from zero to r . Field is perpendicular to equipotentials and points towards lower potential. Potential value and potential gradient differ: opposing sources can cancel signed potentials while their fields add, or a reference can assign zero potential where the gradient is nonzero.

3.2 Neglect other forces and balance electrical force upwards against weight. A downwards field requires negative charge for upwards force. Use $|q|E = mg$.

$$|q| = \frac{mg}{E} = \frac{6.52 \times 10^{-15} \text{ kg} \times 9.81 \text{ m s}^{-2}}{2.00 \times 10^5 \text{ N C}^{-1}} = 3.20 \times 10^{-19} \text{ C}$$

Signed charge $-3.20 \times 10^{-19} \text{ C}$.

$$n = \frac{|q|}{e} = \frac{3.20 \times 10^{-19} \text{ C}}{1.60 \times 10^{-19} \text{ C}} = 2.00$$

Two excess electrons.

3.3 Set threshold electrical force equal to weight using sphere volume.

$$qE = \rho \frac{4\pi r^3}{3} g \quad \Rightarrow \quad d_{\text{max}} = 2 \left(\frac{3qE}{4\pi \rho g} \right)^{1/3}$$

Rearrangement and diameter factor.

$$d_{\text{max}} = 2 \left(\frac{3qE}{4\pi \rho g} \right)^{1/3} = 2 \left(\frac{3 \times 8.0 \times 10^{-15} \text{ C} \times 1.0 \times 10^5 \text{ N C}^{-1}}{4\pi \times 2500 \text{ kg m}^{-3} \times 9.81 \text{ m s}^{-2}} \right)^{1/3} = 3.96 \times 10^{-5} \text{ m}$$

Magnitude $39.6 \mu\text{m}$. Only $30 \mu\text{m}$ is below threshold; 40 is not justified by rounding the threshold up. On the tilt, normal detachment requires $qE > mg \cos 30^\circ$, while the weight also has a downslope component and adhesion is still excluded.