4.1 Impulse and two-dimensional collisions

Name: _____ Class: _____ Date: _____ **Total: 31 marks**

KEY WORDS Momentum • Impulse • isolated system • elastic collision • inelastic collision • non-relativistic particle

Objective

Develop the methods for impulse and two-dimensional collisions before the curated WPH14 questions. Use $g = 9.81$ m s^{-2} where needed. These are original training questions, not copied past-paper questions.

1 Key knowledge and worked examples

■ Force changes momentum

Momentum is the vector $\mathbf{p} = m\mathbf{v}$. Choose a positive direction before using signs. **Impulse** is the change in momentum. For constant force, or the average force over an interval:

$$F_{\text{average}}\Delta t = \Delta p = m(v - u)$$

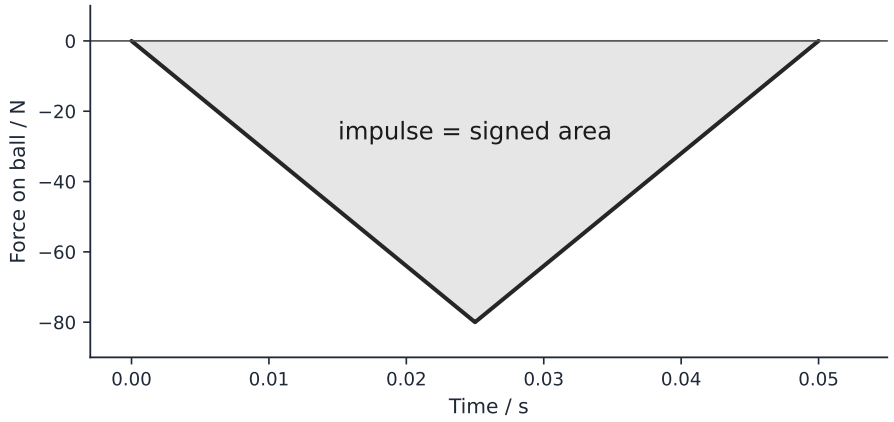
A changing force gives an impulse equal to the signed area under its force–time graph. A triangular pulse has area $\frac{1}{2} \times \text{base} \times \text{height}$. An area below the time axis is negative. The maximum force is not automatically the average force.

Worked example. A 0.20 kg ball approaches a wall at $+6.0$ m s^{-1}. It rebounds at -4.0 m s^{-1}. Contact lasts 0.050 s. Find average force on the ball.

- Known: mass, signed initial/final velocities and contact time.
- Why: force impulse changes the ball's momentum; rebound reverses the sign.

$$\Delta p = m(v - u) = 0.20 \text{ kg} \times (-4.0 - 6.0) \text{ m s}^{-1} = -2.0 \text{ kg m s}^{-1}$$

$$F_{\text{average}} = \frac{\Delta p}{\Delta t} = \frac{-2.0 \text{ kg m s}^{-1}}{0.050 \text{ s}} = -40 \text{ N}$$



The force is opposite the chosen positive direction. The wall feels an opposite force. A deforming bumper or helmet increases stopping time. For the same momentum change, this reduces average force. It does not remove the momentum change.

■ Conserve each momentum component

An **isolated system** has no significant external impulse during the event. Its total momentum is conserved. Internal forces cancel in opposite pairs. Momentum conservation does not require kinetic energy conservation.

For a collision in a plane, resolve every momentum into two perpendicular axes:

$$\sum p_{x,\text{before}} = \sum p_{x,\text{after}} \qquad \sum p_{y,\text{before}} = \sum p_{y,\text{after}}$$

Use the angle measured from the chosen axis. Include signs; a downward component is negative if upwards is positive. Do not add momentum magnitudes when directions differ.

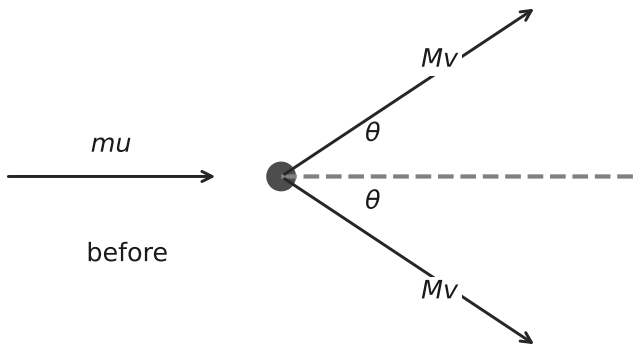
Worked example. A 0.10 kg sphere moving at 2.0 m s^{-1} strikes two stationary spheres, each 0.10 kg. It stops. The two others leave symmetrically at 1.2 m s^{-1}, at angles $+\theta$ and $-\theta$ to the original direction. Find their separation angle and classify the collision.

- Why: transverse momenta cancel by symmetry; longitudinal momentum gives θ.

$$mu = 2Mv\cos\theta$$

$$\cos\theta = \frac{mu}{2Mv} = \frac{0.10 \text{ kg} \times 2.0 \text{ m s}^{-1}}{2 \times 0.10 \text{ kg} \times 1.2 \text{ m s}^{-1}} = 0.833$$

$$2\theta = 2\cos^{-1}\left(\frac{mu}{2Mv}\right) = 2\cos^{-1}(0.833) = 67.1^\circ$$



An **elastic collision** conserves total kinetic energy as well as momentum. An **inelastic collision** does not conserve kinetic energy; some becomes other forms of energy.

$$E_{k,\text{before}} = \frac{1}{2}mu^2 = \frac{1}{2} \times 0.10 \times 2.0^2 \text{ J} = 0.20 \text{ J}$$

$$E_{k,\text{after}} = 2 \times \frac{1}{2}Mv^2 = 2 \times \frac{1}{2} \times 0.10 \times 1.2^2 \text{ J} = 0.144 \text{ J}$$

This collision is inelastic. Total energy is still conserved. In an ordinary passive collision, kinetic energy cannot increase without an additional energy source.

For a **non-relativistic particle** , use $p = mv$ and $E_k = \frac{1}{2}mv^2$. Substituting $v = p/m$ gives:

$$E_k = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \frac{p^2}{2m}$$

At equal kinetic energy, $p = \sqrt{2mE_k}$: a larger mass has larger momentum. At equal momentum, a larger mass has smaller kinetic energy. These equations are not valid for a particle moving close to light speed.

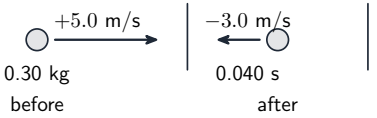
■ Core practicals 9 and 10

For **core practical 9**, use a trolley, force sensor and data logger. Measure trolley mass and velocities just before/after contact using light gates or a motion sensor. Record the force–time pulse at a sampling rate fast enough to resolve it. Zero the force sensor; use consistent axes for force and velocity. Integrate the graph area and compare it with $m(v - u)$. Repeat for different contact forces or speeds. A graph of impulse against momentum change should have gradient close to one. Friction, sensor offsets and missed parts of the pulse can cause disagreement. Secure the track and catch the trolley safely.

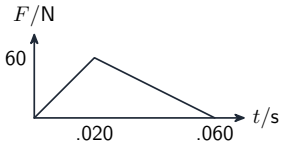
For **core practical 10**, record small spheres colliding on a level surface using an overhead camera. Include a length scale in the collision plane and a known frame rate. Keep the camera perpendicular to reduce perspective error. Track centres over several frames before and after the short collision. Convert pixel displacements to metres and frame intervals to seconds. Determine both velocity components, then compare total momentum components and kinetic energies. Repeat with different approach directions. Include uncertainty from positions, timing and scale calibration. A small mismatch within uncertainty is not evidence that momentum fails. Keep spheres contained so they cannot fall or become a slipping hazard.

2 Practice

2.1 A 0.30 kg ball moves at $+5.0$ m s^{-1} and rebounds at -3.0 m s^{-1} after 0.040 s of contact. Calculate its impulse and average force, stating directions. [4]



2.2 A force pulse rises linearly from zero to 60 N in 0.020 s and falls linearly to zero at 0.060 s. It acts in the positive direction on a 0.40 kg trolley initially at rest. Find impulse and final speed. Explain why using 60 N as average force is wrong. [5]

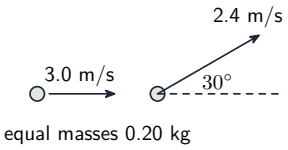


2.3 Two non-relativistic particles have equal kinetic energy. Their masses are m and $4m$. Derive an expression for kinetic energy in terms of momentum and mass, then find the ratio of their momenta, heavier to lighter. [3]

m $4m$ equal E_k

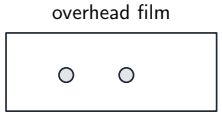
3 Exam-style questions

3.1 A 0.20 kg puck moving at 3.0 m s^{-1} along $+x$ strikes an identical stationary puck. Afterwards the first moves at 2.4 m s^{-1} at $+30.0°$ to x. Find the second puck's two velocity components and its speed and direction. Test whether the collision is elastic. Assume negligible external impulse. [7]



3.2 Describe how a force sensor, light gates and data logger can test impulse equals momentum change for a trolley collision. Include processing, a sampling issue, a cause of systematic disagreement and one safety measure. [6]

3.3 A student films two spheres colliding and finds momentum conserved to within 3% but kinetic energy apparently increases by 4%. Explain how to obtain the two velocity components from the film, identify two calibration or recording controls, and explain why the reported percentages alone cannot establish a real energy gain. [6]



Solutions

Original-task marks: M1 = method, A1 = accuracy, B1 = stated reasoning point.

2.1 Rebound reverses the velocity sign.

$$\Delta p = m(v-u) = 0.30\ \text{kg} \times (-3.0 - 5.0)\ \text{m s}^{-1} = -2.4\ \text{kg m s}^{-1}$$

Impulse is negative, opposite the approach. Use impulse per contact time.

$$F_{\text{av}} = \frac{\Delta p}{\Delta t} = \frac{-2.4\ \text{kg m s}^{-1}}{0.040\ \text{s}} = -60\ \text{N}$$

Average force is 60 N opposite approach.

2.2 Integrate the triangular pulse.

$$J = \frac{1}{2} F_{\max} \Delta t = \frac{1}{2} \times 60\ \text{N} \times 0.060\ \text{s} = 1.8\ \text{N s}$$

Impulse +1.8 N s. Use $J = m(v-u)$ with $u = 0$.

$$v = \frac{J}{m} = \frac{1.8\ \text{N s}}{0.40\ \text{kg}} = 4.5\ \text{m s}^{-1}$$

Final speed. The force is below its maximum for almost all the interval; its average here is half the peak.

2.3 Rearrange momentum to eliminate velocity.

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \frac{p^2}{2m}$$

At equal energy, $p \propto \sqrt{m}$, so heavier:lighter is $2:1$.

3.1 Resolve the momentum in both axes; equal masses cancel.

$$v_{2x} = u - v_1 \cos\theta = 3.0\ \text{m s}^{-1} - 2.4\ \text{m s}^{-1} \cos 30.0^\circ = 0.922\ \text{m s}^{-1}$$

x component.

$$v_{2y} = -v_1 \sin\theta = -2.4\ \text{m s}^{-1} \sin 30.0^\circ = -1.20\ \text{m s}^{-1}$$

y component. Recombine perpendicular components.

$$v_2 = \sqrt{v_{2x}^2 + v_{2y}^2} = \sqrt{0.922^2 + 1.20^2}\ \text{m s}^{-1} = 1.51\ \text{m s}^{-1}$$

$$\beta = \tan^{-1}\left(\frac{|v_{2y}|}{v_{2x}}\right) = \tan^{-1}\left(\frac{1.20}{0.922}\right) = 52.5^\circ$$

Speed and direction 52.5° below $+x$. Compare total kinetic energies.

$$E_{k,b} = \frac{1}{2}mu^2 = \frac{1}{2} \times 0.20\ \text{kg} \times (3.0\ \text{m s}^{-1})^2 = 0.900\ \text{J}$$

$$E_{k,a} = \frac{1}{2}m(v_1^2 + v_2^2) = \frac{1}{2} \times 0.20\ \text{kg} \times (2.4^2 + 1.513^2)\ \text{m}^2\text{s}^{-2} = 0.805\ \text{J}$$

Kinetic energy decreases: inelastic.

3.2 Measure trolley mass and use light gates with a measured interrupt length for velocities immediately before and after. Zero the force sensor and use consistent positive directions for force and velocity. Integrate the entire force–time pulse, including signed areas. Compare that impulse with $m(v-u)$ over repeated trials. Choose high enough sampling rate to resolve the short pulse; a missed peak, sensor offset or friction can cause systematic disagreement. Secure the track and catch the trolley to prevent falls.

3.3 Track sphere centres over several frames before and after contact and divide calibrated x and y displacements by the time interval. Put a length scale in the collision plane. Use a known frame rate and camera perpendicular to the plane to reduce perspective error. Calculate kinetic energies from both components and measured masses, propagating or estimating uncertainty. A 4% increase may lie within the combined energy uncertainty, especially because velocity is squared. For a passive collision, investigate calibration/tracking errors or an additional energy source before claiming a physical increase; agreement of momentum to 3% does not by itself validate energy to 4%.