3.3 Graphs and justified conclusions

Name: _____ Class: _____ Date: _____ **Total: 28 marks**

KEY WORDS best-fit line • gradient • intercept • error bar • logarithmic graph

Objective

Build the skills to answer exam questions on **graphs and justified conclusions** : choosing axes for a straight line, scales and best-fit lines, gradients and intercepts, logarithmic graphs, error bars and worst-fit lines.

You must be able to:

- choose quantities to plot to test a relationship
- find a gradient from a large triangle and link it to a physical constant
- use a log graph to find the power n in $y = kx^n$
- use error bars and a worst-fit line to find the uncertainty in a gradient

1 Key knowledge and worked examples

From graph to conclusion

idea	rule
scales	the points fill more than half the grid; easy steps (1, 2, 5)
best-fit line	even scatter of points on each side; leave out clear anomalies
gradient	use a triangle that covers more than half the line
power law	$y = kx^n$ gives $\lg y = n \lg x + \lg k$: gradient n
exponential	$y = y_0 e^{-\lambda x}$ gives $\ln y = -\lambda x + \ln y_0$: gradient $-\lambda$
uncertainty in a gradient	$\dfrac{\lvert\text{worst gradient} - \text{best gradient}\rvert}{\text{best gradient}} \times 100\%$

Worked example 1: a straight line. A mass on a spring has period $T = 2\pi\sqrt{m/k}$. What should be plotted, and what does the gradient give?

1. Known: k is the constant to find.
2. Rule: square both sides: $T^2 = \left(\dfrac{4\pi^2}{k}\right) m$.
3. Plot: T^2 against m.
4. Result: **gradient** $= 4\pi^2/k$**, so** $k = 4\pi^2/$**gradient**.

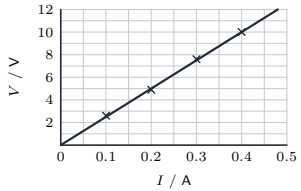
Worked example 2: a log graph. A graph of $\lg y$ against $\lg x$ passes through $(0.20, 0.90)$ and $(1.20, 3.90)$. Find n in $y = kx^n$.

1. Known: two points on the line.
2. Rule: n = gradient.
3. Numbers: $(3.90 - 0.90)/(1.20 - 0.20)$.
4. Result: $n = 3.0$.

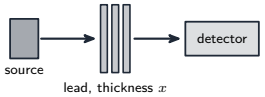
Answer shapes: a conclusion says what the graph shows (straight line? through the origin?) and gives the value with its unit and uncertainty.

2 Practice

2.1 The graph shows the p.d. across a resistor against the current. Use the best-fit line to find the resistance. [2]



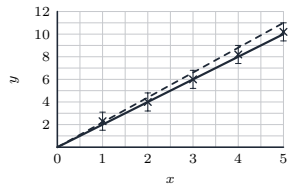
2.2 The intensity of gamma rays through lead of thickness x is expected to follow $I = I_0 e^{-\mu x}$. State what the student should plot to get a straight line, and how to find μ and I_0 from it. [3]



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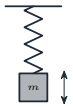
3 Exam-style questions

3.1 The graph shows results with error bars, the best-fit line (solid) and the steepest acceptable line (dashed). Find the best gradient and its percentage uncertainty. [4]



3.2 A student measures the period T of a mass m oscillating on a spring. Her results are in the table.

(a) Explain why plotting T^2 against m tests the equation $T = 2\pi\sqrt{m/k}$, and what the graph should look like if it is true. [3]



m / kg	0.10	0.20	0.30	0.40	0.50
T / s	0.63	0.89	1.09	1.26	1.40

(b) Calculate T^2 for $m = 0.10$ kg and $m = 0.50$ kg. Use these values to find the gradient and the spring constant k. [4]

(c) Another student does not know the equation. She suggests $T = am^n$. Explain what she should plot, and use the values for $m = 0.10$ kg and $m = 0.50$ kg to find n. State what your value shows. [6]

(d) The real graph of T^2 against m is a straight line with a small positive intercept. Explain what this could show, and how the student should justify her conclusion. [6]

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Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

- gradient $= 10.0/0.40$
- $R = \mathbf{25}\,\Omega$

2.2

- plot $\ln I$ against x
- gradient $= -\mu$, so $\mu = -$gradient
- intercept on the $\ln I$ axis $= \ln I_0$, so $I_0 = e^{\text{intercept}}$

3.1

- best gradient $= 10.0/5.0 = \mathbf{2.0}$
- worst gradient $= 11.0/5.0 = 2.2$
- uncertainty $= \dfrac{0.2}{2.0} \times 100\% = \mathbf{10}\%$

3.2

(a) squaring gives $T^2 = \left(\dfrac{4\pi^2}{k}\right) m$; this is $y = mx$, so the graph should be a straight line through the origin

(b) $T^2 = 0.63^2 = 0.40\ \text{s}^2$ and $1.40^2 = 1.96\ \text{s}^2$; gradient $= (1.96 - 0.40)/0.40 = 3.9\ \text{s}^2\ \text{kg}^{-1}$; $k = 4\pi^2/3.9 = \mathbf{10}\ \text{N m}^{-1}$

(c) $\lg T = n \lg m + \lg a$, so plot $\lg T$ against $\lg m$; the gradient is n; points: $(\lg 0.10, \lg 0.63) = (-1.00, -0.20)$ and $(\lg 0.50, \lg 1.40) = (-0.30, 0.15)$; $n = 0.35/0.70 = \mathbf{0.50}$; so T is proportional to $\sqrt{m}$, as the equation predicts

(d) points to include:

- the intercept means T is not zero when the added mass is zero
- the spring itself has mass, which also oscillates, so the real moving mass is bigger than m
- a systematic error (for example a zero error, or timing too few oscillations) could also give an intercept
- the line is straight, so T^2 depends on m in a linear way: the gradient still gives k
- check whether the intercept is within the error bars, using worst-fit lines
- give k with its uncertainty from the best and worst gradients, and say if the equation is supported

Levels: **5–6** a full, logical answer with correct physics and correct terms; **3–4** several correct points, partly linked; **1–2** one or two relevant points.

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