

2.2 Superposition and standing waves

Name: _____ Class: _____ Date: _____ Total: 26 marks

KEY WORDS wavefront • Phase • Superposition • Interference
 • Coherent sources • path difference • standing wave • node
 • antinode • tension • mass per unit length • two-beam oscilloscope

Objective

Build the methods for WPH12 statements 38–43: superposition and standing waves. These are original teaching questions, not copied past-paper items.

1 Key knowledge and worked examples

A **wavefront** joins points at the same phase. **Phase** describes the stage of a vibration cycle. One complete cycle corresponds to 360° or 2π radians.

Superposition means adding the displacements of overlapping waves at each point. **Interference** is the resulting reinforcement or cancellation. **Coherent sources** have the same frequency and a constant phase difference. Their phase difference need not be zero.

For waves of the same wavelength, the phase difference caused by a **path difference** Δx is:

$$\Delta\phi = \frac{\Delta x}{\lambda} 360^\circ = \frac{\Delta x}{\lambda} 2\pi \text{ rad}$$

For sources initially in phase, path differences $0, \lambda, 2\lambda, \dots$ give constructive interference. Half-integer wavelength differences give destructive interference. Complete cancellation also needs equal amplitudes. If sources start with a phase difference, include it too.

Worked example. Two in-phase speakers produce wavelength 0.80 m. Their paths to a microphone differ by 1.20 m.

- Known: path difference and wavelength. Convert path difference into cycles, then phase.

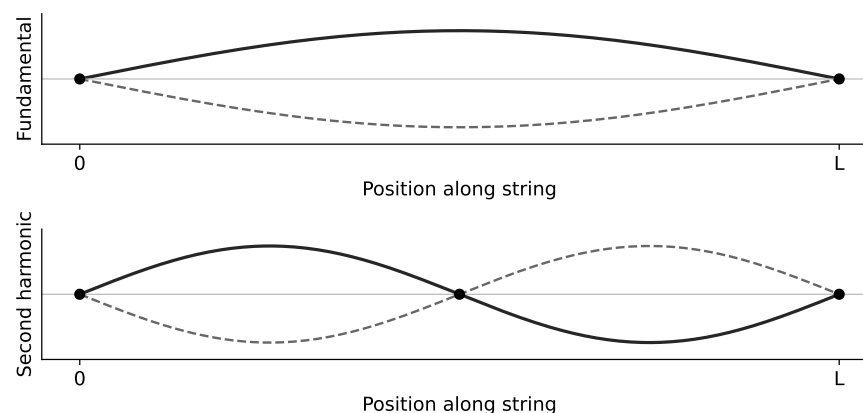
$$\Delta\phi = \frac{\Delta x}{\lambda} 360^\circ = \frac{1.20}{0.80} 360^\circ = 540^\circ$$

- 540° is equivalent to 180° . The waves arrive in opposite phase and interfere destructively. The sound need not become silent if the arriving amplitudes differ.

■ How a standing wave forms

A **standing wave** forms when two waves of the same frequency and similar amplitude travel in opposite directions and superpose. Reflection at a fixed string end supplies the returning wave.

- A **node** always has zero displacement. An **antinode** has the largest vibration amplitude.
- Adjacent nodes are $\lambda/2$ apart. A node and its nearest antinode are $\lambda/4$ apart.
- Points between the same pair of nodes vibrate in phase. Points in neighbouring sections vibrate in opposite phase.
- All points have the same frequency, except that a node does not vibrate. Amplitude depends on position.
- An ideal standing wave has no net energy transfer along it. A travelling wave carries energy along its direction of travel.



The end nodes remain still; the curve changes between the solid and dashed shapes.

For a string fixed at both ends, length L contains a whole number k of half-wavelengths:

$$L = \frac{k\lambda}{2} \quad v = \sqrt{\frac{F_T}{\mu}} \quad f = \frac{k}{2L} \sqrt{\frac{F_T}{\mu}}$$

Here F_T is **tension**, not period; μ is **mass per unit length** in kg m^{-1} . The fundamental has $k = 1$. At fixed mode, frequency increases with $\sqrt{F_T}$, decreases with L , and decreases with $\sqrt{\mu}$.

Worked example. A 0.60 m string has $\mu = 1.5 \times 10^{-3} \text{ kg m}^{-1}$ and tension 24 N. Find its fundamental frequency.

- Known: length, tension and linear density. The fundamental fits half a wavelength between the fixed ends.

$$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{24}{1.5 \times 10^{-3}}} \text{ m s}^{-1} = 126 \text{ m s}^{-1}$$

$$f = \frac{v}{2L} = \frac{126 \text{ m s}^{-1}}{2 \times 0.60 \text{ m}} = 105 \text{ Hz}$$

- Doubling tension multiplies frequency by $\sqrt{2}$, rather than by two.

■ Core practical 4: speed of sound

Use a signal generator, speaker, movable microphone and **two-beam oscilloscope**. Connect one oscilloscope channel to the generator reference signal and the other to the microphone output. Both traces have the same frequency.

- Keep frequency constant. Move the microphone along a measured line away from the speaker. Find a position where the traces are in phase.
- Move to another in-phase position. Consecutive in-phase positions are one wavelength apart. Measure across several wavelengths and divide by their number.
- Read the period from the time-base scale: count horizontal divisions across several cycles, then divide. Find $f = 1/T$ and use $v = f\lambda$.
- Repeat positions and frequencies. Keep the microphone on the speaker axis. Reduce reflected sound by working away from walls. Measure position from a fixed microphone reference point.

Measuring several wavelengths reduces the percentage uncertainty in the distance. Measuring several periods does the same for time. A resonance-tube experiment can also measure sound speed, but it does not replace this specified practical.

■ Core practical 5: vibrating string

Use a vibration generator driven by a signal generator. Run the string over a pulley to a hanging mass. The tension is about mg when pulley friction is small. Increase frequency until a clear standing-wave pattern forms.

- Investigate tension: keep L , μ and mode fixed. Vary hanging mass. Plot f^2 against F_T ; the gradient for the fundamental is $1/(4L^2\mu)$.
- Investigate length: keep tension, string type and mode fixed. Plot f against $1/L$.
- Investigate linear density: measure string mass and length, $\mu = m/l$. Use different strings with fixed tension, vibrating length and mode. Plot f^2 against $1/\mu$.
- Measure length between the end nodes. Find the resonance from both higher and lower frequencies and repeat. Secure the stand and keep clear of falling masses.

■ Standing sound in a closed tube

In the simplest standing sound wave in a tube closed at one end, the closed end is a molecular displacement node. The open end is approximately a displacement antinode. This fits one-quarter wavelength into the effective tube length, so

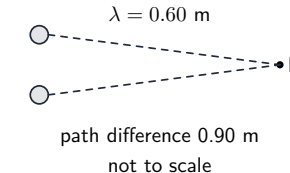
$L_{\text{effective}} = \lambda/4$. The pressure pattern is reversed: the closed end is a pressure antinode. Do not call a pressure node a displacement node. Real tubes have a small open-end correction; use an effective length if the question supplies it.

Worked example. A closed tube has effective length 0.20 m and fundamental frequency 425 Hz. The end conditions give $\lambda = 4L = 0.80$ m; then $v = f\lambda = 425 \times 0.80 = 340 \text{ m s}^{-1}$. A string fixed at both ends instead fits half a wavelength in its fundamental. Choose from the end conditions, not from the word “standing”.

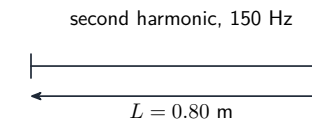
2 Practice

2.1 Define coherent sources. Must coherent sources always vibrate in phase? [2]

2.2 Two in-phase sources have wavelength 0.60 m. Their path difference at a point is 0.90 m. Calculate phase difference and describe the interference. State a further condition for complete cancellation. [3]



2.3 A 0.80 m string fixed at both ends vibrates in its second harmonic at 150 Hz. Find wavelength and speed. State the separation of adjacent nodes and the phase relation between neighbouring loops. [4]



3 Exam-style questions

3.1 A string has $L = 0.50$ m, $\mu = 2.0 \times 10^{-3}$ kg m⁻¹ and tension 18 N. Calculate its fundamental frequency. Predict the frequency if tension becomes 72 N. Explain how to investigate tension without confusing its effect with another variable. [5]

$$F_T = 18 \text{ N}$$

$$L = 0.50 \text{ m}, \mu = 2.0 \times 10^{-3} \text{ kg/m}$$

3.2 Plan the specified two-beam oscilloscope measurement of sound speed. Include the two signals, wavelength measurement, frequency measurement, calculation and one way to reduce percentage distance uncertainty. [6]

3.3 For a fixed-end string, explain how to investigate separately the effects of length and mass per unit length on fundamental frequency. Give a suitable graph for each investigation and two controls for each. [6]

Solutions

These allocations are for the original questions on this sheet. B1 = one standalone point, M1 = one method point, A1 = one accuracy point.

2.1 They have the same frequency and a constant phase difference. No: that constant phase difference can be non-zero.

2.2 $\Delta\phi = (\Delta x/\lambda)360^\circ = (0.90/0.60)360^\circ = 540^\circ$, equivalent to 180° . Destructive interference. Equal arriving amplitudes are also needed for complete cancellation.

2.3 The second harmonic has two half-wavelengths in L : $\lambda = L = 0.80$ m. $v = f\lambda = 150 \times 0.80 = 120$ m s⁻¹. Node separation = $\lambda/2 = 0.40$ m. Neighbouring loops vibrate in opposite phase.

3.1 For the fundamental, $\lambda = 2L$. $v = \sqrt{F_T/\mu} = \sqrt{18/0.002} = 94.9$ m s⁻¹. $f = v/(2L) = 94.9$ Hz. Quadrupling tension doubles f to 190 Hz. Keep string linear density, vibrating length and harmonic number fixed while varying tension.

3.2 Connect a generator reference and microphone output to separate channels; the generator drives a speaker. Move the microphone along a measured line and identify repeated in-phase positions. Separation of consecutive in-phase positions gives λ . Read period from divisions and time base, then $f = 1/T$. Calculate $v = f\lambda$. Measure across several wavelengths and divide by their count, reducing percentage distance uncertainty.

3.3 Length investigation: vary vibrating length and tune to the fundamental each time, keeping tension and string linear density fixed. Plot f against $1/L$ for a straight line through the origin. Density investigation: use strings with different measured $\mu = m/l$ and tune each to the fundamental, keeping tension and vibrating length fixed. Plot f^2 against $1/\mu$ for a straight line through the origin.