1.1 Motion graphs and uniform acceleration

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS displacement • velocity • acceleration • gradient • tangent • uniform acceleration • systematic error • light gate

Objective

Build the skills to answer WPH11 questions on **motion graphs** and **uniformly accelerated motion**: read gradients and areas, choose the right suvat equation, and finish every "deduce" question with a compared conclusion. This sheet covers Unit 1 statements 1–3 and core practical 1.

You must be able to:

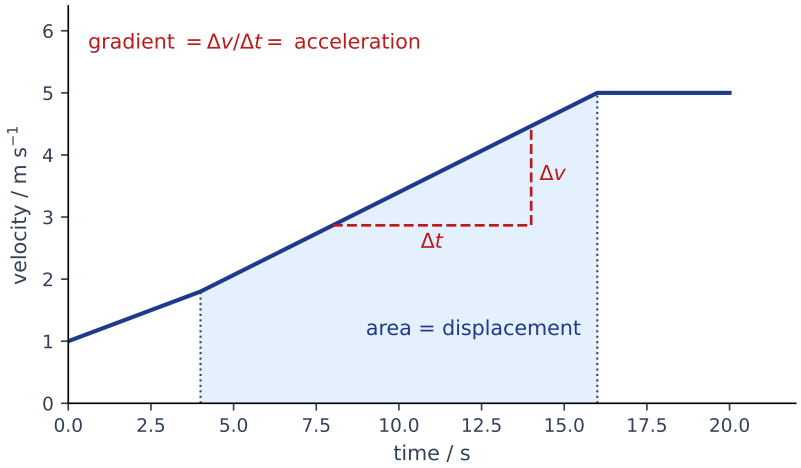
- read acceleration from the gradient of a velocity–time graph and displacement from its area
- tell distance from displacement when areas have both signs
- draw and use a tangent on a curved graph, and estimate an area by counting squares
- choose between the four suvat equations by the quantities given
- finish *deduce whether…* questions with a comparison **and** a conclusion
- describe how to determine the acceleration of free fall, including the s–t^2 graph

1 Key knowledge and worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

■ Gradient and area

On a velocity–time graph the **gradient** is the acceleration and the **area** between the line and the time axis is the displacement. Area below the axis is negative displacement; the distance adds the sizes of all the areas.



Worked example. Find the maximum acceleration shown on the graph.

- Known: the steepest straight part runs from (4 s, 1.8 m s^{-1}) to (16 s, 5.0 m s^{-1}). Why: acceleration is the gradient of a velocity–time graph.

$$a = \frac{\Delta v}{\Delta t}$$

$$a = \frac{5.0\ \text{m s}^{-1} - 1.8\ \text{m s}^{-1}}{16\ \text{s} - 4\ \text{s}} = 0.27\ \text{m s}^{-2}$$

- Check: read both values from the line itself, not from grid lines nearby; the unit is m s^{-2}.

■ Choosing a suvat equation

The four equations connect s, u, v, a, t for **constant** acceleration:

$$v = u + at \qquad s = \frac{(u+v)t}{2} \qquad s = ut + \tfrac{1}{2}at^2 \qquad v^2 = u^2 + 2as$$

given	wanted	use
u, a, t	v	$v = u + at$
u, v, t	s	$s = \frac{(u+v)}{2}t$
u, a, t	s	$s = ut + \frac{1}{2}at^2$
u, v, a	s (no time!)	$v^2 = u^2 + 2as$

Worked example (deduce-and-compare). A ball leaves a bat at 12 m s^{-1} upwards. Deduce whether it rises more than 6.0 m. Take upwards as positive.

- Known: $u = +12$ m s^{-1}, $v = 0$ at the top, $a = -9.81$ m s^{-2}. No time, so use $v^2 = u^2 + 2as$.

$$v^2 = u^2 + 2as \quad \Rightarrow \quad s = \frac{v^2 - u^2}{2a}$$

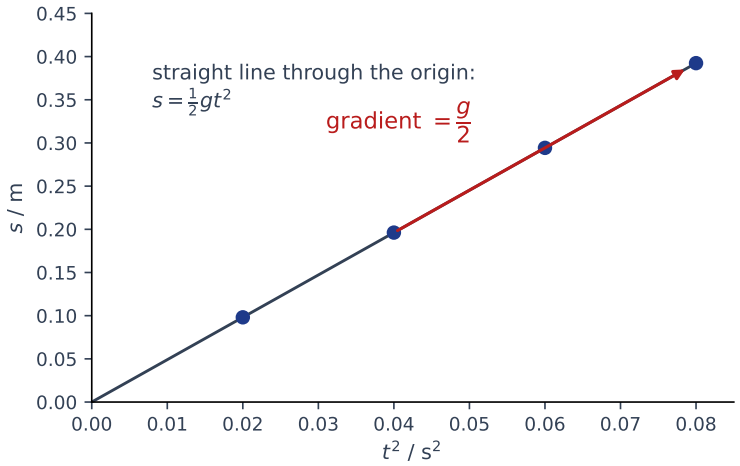
$$s = \frac{0 - (12\ \text{m s}^{-1})^2}{2 \times (-9.81\ \text{m s}^{-2})} = \frac{-144}{-19.62}\ \text{m} = 7.3\ \text{m}$$

- $7.3\ \text{m} > 6.0\ \text{m}$, so the ball **does** rise more than 6.0 m. The comparison and the conclusion earn the last mark.

■ Core practical 1: the acceleration of free fall

Drop a ball through two light gates (or film it against a ruler): measure the fall distance s and the time t, or the velocities at the gates. Then:

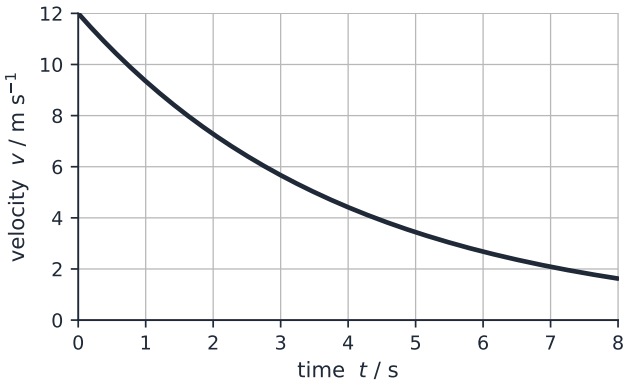
- one pair of values: $s = ut + \frac{1}{2}at^2$ with $u = 0$ gives $g = \dfrac{2s}{t^2}$;
- better: several fall distances, plotted as s against t^2. Since $s = \frac{1}{2}gt^2$, the graph is a straight line through the origin with gradient $g/2$, so $g = 2 \times$ gradient.



A straight line that misses the origin shows a **systematic error**, for example a constant distance offset. A timing delay can distort the relationship rather than merely changing its intercept. Repeating drops reduces random scatter; it cannot remove an offset. In a strobe photograph, a shorter flash means the ball travels less distance during the flash. The image is sharper and the position reading has less uncertainty, if the scale and image resolution are unchanged.

■ Curves: tangents and squares

When a velocity–time graph is curved, the acceleration **changes**: find the acceleration at one instant from the gradient of a tangent, and estimate the area by counting grid squares (count a square when at least half of it lies under the curve).



Worked example. Estimate the acceleration of the cyclist at $t = 2.0$ s.

- Known: the curve; a tangent at $t = 2.0$ s passes through about (0, 10.8) and (4.0 s, 4.4 m s^{-1}). Why: the tangent's gradient is the instantaneous acceleration.

$$a = \frac{\Delta v}{\Delta t} = \frac{4.4\ \text{m s}^{-1} - 10.8\ \text{m s}^{-1}}{4.0\ \text{s} - 0} = -1.6\ \text{m s}^{-2}$$

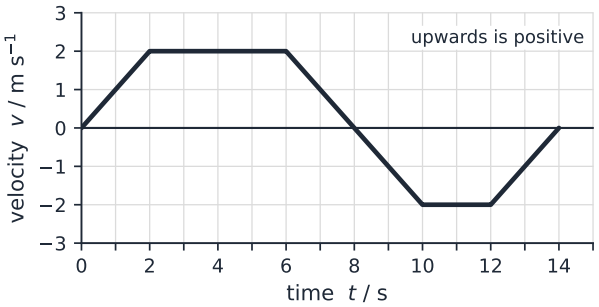
- Check: the curve is steepest at the start, so the tangent gradient here (1.6 m s^{-2} in size) should be larger than the average gradient over the whole 8 s (1.3 m s^{-2}) – it is.

2 Practice

Now use the methods above.

2.1 State what is represented by (a) the gradient of a velocity–time graph; (b) the area between the line and the time axis on a velocity–time graph. [2]

2.2 The velocity–time graph shows a lift that goes up, comes back down, and passes its starting floor. Upwards is positive.



(a) Calculate the acceleration of the lift between $t = 0$ and $t = 2.0$ s. [1]

(b) Calculate the displacement of the lift between $t = 6.0$ s and $t = 12.0$ s. [2]

(c) Calculate the total distance travelled by the lift in the 14 s. *(Hint: for distance, every area counts as positive.)* [2]

2.3 A ball is dropped from rest and hits the ground after 0.55 s.

(a) Use $s = ut + \frac{1}{2}at^2$ to show that the drop height is about 1.5 m. [2]

(b) State one reason why a measured time from a handheld stopwatch could make the calculated g too small, and one improvement. [2]

2.4 A car accelerates uniformly from 8.0 m s^{-1} to 20 m s^{-1} over 96 m. Calculate its acceleration. [3]

3 Exam-style questions

3.1 An object moves with constant non-zero acceleration. Which row describes its displacement–time graph and its velocity–time graph? [1]

☐ **A** both graphs are straight lines

B the displacement–time graph is curved; the velocity–time graph is a straight sloped line

C both graphs are curved

D the displacement–time graph is a straight sloped line; the velocity–time graph is curved

3.2 A train's velocity–time graph is a straight line from 3.0 m s^{-1} at $t = 0$ to 9.0 m s^{-1} at $t = 4.0$ s. What is the displacement in those 4.0 s? [1]

☐ **A** 12 m

B 24 m

C 36 m

D 48 m

3.3 In an experiment to determine g, a student plots s on the y-axis for a falling object.

(a) State what should be plotted on the x-axis to give a straight line through the origin, and how the gradient gives g. [2]

(b) The student's s against t^2 line has a small fixed vertical intercept but the expected gradient. Suggest a distance-measurement error that could cause this and state the effect on g calculated from the gradient. [2]

3.4 A skydiver leaves an aeroplane with zero vertical velocity. In the first 1.4 s she falls 9.2 m. Air resistance can be ignored if her mean acceleration is greater than $0.95\,g$. Deduce whether air resistance can be ignored. [4]

Solutions

2.1 (a) acceleration (b) displacement. *Distance needs the sizes of the areas; displacement uses their signs.*

2.2 (a) $a = \dfrac{\Delta v}{\Delta t} = \dfrac{2.0\ \text{m s}^{-1} - 0}{2.0\ \text{s}} = 1.0\ \text{m s}^{-2}$. (b) Areas: $t = 6$–8 s: $\frac{1}{2} \times 2.0\ \text{s} \times 2.0\ \text{m s}^{-1} = +2.0$ m; $t = 8$–10 s: $\frac{1}{2} \times 2.0\ \text{s} \times (-2.0\ \text{m s}^{-1}) = -2.0$ m; $t = 10$–12 s: $2.0\ \text{s} \times (-2.0\ \text{m s}^{-1}) = -4.0$ m. Displacement $= 2.0 - 2.0 - 4.0 = -4.0$ m. (c) Distance $= |+2.0| + |-2.0| + |-4.0|$ plus the 0–6 s area $\frac{1}{2}(4.0+6.0)\ \text{s} \times 2.0\ \text{m s}^{-1} = 10$ m and the 12–14 s triangle $\frac{1}{2} \times 2.0\ \text{s} \times 2.0\ \text{m s}^{-1} = 2.0$ m: total $10 + 2.0 + 2.0 + 4.0 + 2.0 = 20$ m.

2.3 (a) $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 9.81\ \text{m s}^{-2} \times (0.55\ \text{s})^2 = 1.5$ m. (b) Reaction time makes t too long, so $g = 2s/t^2$ comes out too small; use light gates or video with a known frame rate instead.

2.4 No time is given, so $v^2 = u^2 + 2as$. $a = \dfrac{v^2 - u^2}{2s} = \dfrac{(20\ \text{m s}^{-1})^2 - (8.0\ \text{m s}^{-1})^2}{2 \times 96\ \text{m}} = \dfrac{336}{192}\ \text{m s}^{-2} = 1.8\ \text{m s}^{-2}$.

3.1 B. Constant acceleration makes velocity change linearly (straight sloped velocity–time line) while displacement follows $s = ut + \frac{1}{2}at^2$ (curved). A is wrong because a straight displacement–time line means constant velocity; C and D mismatch the pair.

3.2 B. The area is a trapezium: $s = \frac{(u+v)}{2}t = \frac{3.0+9.0}{2}\ \text{m s}^{-1} \times 4.0\ \text{s} = 24$ m. A uses only u; C multiplies $v \times t$; D doubles correctly counted halves.

3.3 (a) Plot t^2 on the x-axis; $s = \frac{1}{2}gt^2$, so the gradient equals $g/2$ and $g = 2 \times$ gradient. (b) A fixed offset in the distance reading adds the same amount to every s; it changes the vertical intercept but not the gradient, so $g = 2 \times$ gradient is unchanged.

3.4 $s = ut + \frac{1}{2}at^2$ with $u = 0$: $a = \dfrac{2s}{t^2} = \dfrac{2 \times 9.2\ \text{m}}{(1.4\ \text{s})^2} = 9.4\ \text{m s}^{-2}$. Compare: $\dfrac{9.4}{9.81} = 0.96 > 0.95$; since the calculated acceleration exceeds $0.95g$, air resistance **can** be ignored.