Exercise walk-through

4.2
Circular motion (statements 87–91)

Pearson Edexcel · International A-Level · Physics

4.2 Circular motion

Name: _____ Class: _____ Date: _____ **Total: 29 marks**

KEY WORDS radian
- Angular velocity
- tangential speed
- centripetal force

Objective

Develop the methods for circular motion before the curated WPH14 questions. Use $g = 9.81\ \text{m s}^{-2}$ where needed. These are original training questions, not copied past-

paper questions.

1 Key knowledge and worked examples

■ Angle, angular speed and tangential speed

An angle of one **radian** subtends an arc equal to the radius. Hence $\theta = s/r$, with angle in radians. One revolution is 2π radians or 360°.

$$\theta_{\text{rad}} = \theta_{\text{degrees}} \frac{2\pi}{360} \qquad \omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} \qquad v = \omega r$$

Angular velocity describes the rate of angular displacement. In the plane problems here, use its signed rotational rate or magnitude as appropriate. Points on one rigid turntable have the same angular speed. Points farther from its centre have larger **tangential speed** . Their velocity directions also vary around the circle.

Worked example. A wheel rotates at 120 revolutions per minute. Find angular speed and acceleration of a point 0.25 m from its centre.

$$f = \frac{N}{t} = \frac{120}{60\ \text{s}} = 2.0\ \text{Hz}$$

$$\omega = 2\pi f = 2\pi \times 2.0\ \text{s}^{-1} = 12.6\ \text{rad s}^{-1}$$

$$a = r\omega^2 = 0.25\ \text{m} \times (4\pi\ \text{s}^{-1})^2 = 39.5\ \text{m s}^{-2}$$

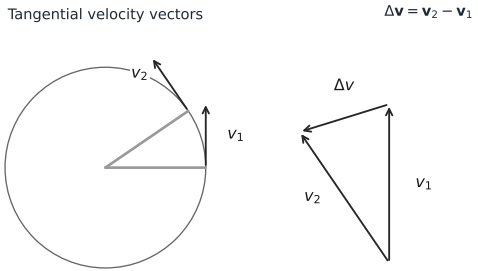
Keep the unrounded angular speed during the calculation. Use radius, not diameter.

Why acceleration points inward

Uniform circular motion has constant speed but changing velocity. Draw the initial and final tangential velocity vectors. The change is $\Delta\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$, not their sum. For a small angular change, the velocity triangle and radius triangle are similar:

$$\frac{\Delta v}{v} \approx \frac{\Delta s}{r} \qquad \Delta s \approx v\Delta t$$

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{v^2}{r} = r\omega^2$$



The change in velocity points towards the centre in this limit. A resultant **centripetal force** is therefore needed:

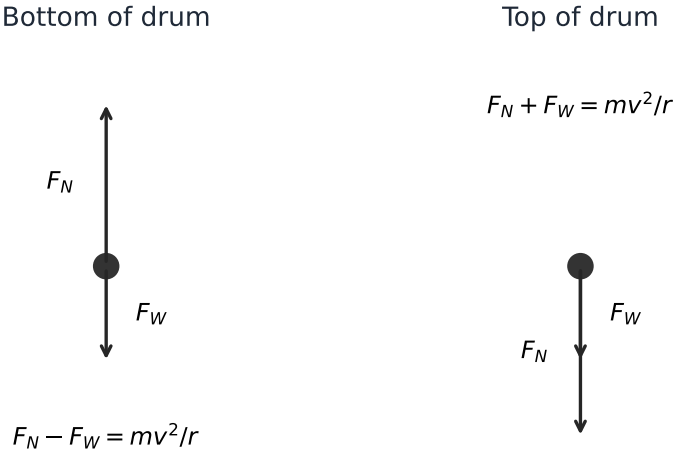
$$\sum F_{\text{inward}} = ma = \frac{mv^2}{r} = mr\omega^2$$

Centripetal force names the resultant of real forces. It is not an extra force to add to a force diagram. Friction can provide it on a turntable. Tension can provide it for a ball on a string.

Write the real-force equation

For clothing against the inside of a vertical drum, weight always points down. The drum's normal reaction points towards the centre. At constant speed:

$$\text{bottom:} \quad F_N - mg = \frac{mv^2}{r} \qquad \text{top:} \quad F_N + mg = \frac{mv^2}{r}$$



The required resultant has the same magnitude, but the reaction is larger at the bottom. If contact is lost, the surface cannot provide a pulling normal reaction. Check this before using a circular-path model.

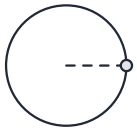
For a horizontal turntable, the largest available friction must be at least $mr\omega^2$. If maximum friction is $mg/25$:

$$mr\omega^2 \leq \frac{mg}{25} \quad \Rightarrow \quad r \leq \frac{g}{25\omega^2}$$

Mass cancels because both required and available forces scale with mass. If the inward force disappears, motion initially follows the tangent. Gravity can then curve the later path; “tangent” describes the release direction.

Exercise 2.1

2.1 A point moves around a circle of radius 0.40 m at 90 revolutions per minute. Calculate angular speed, tangential speed and centripetal acceleration. [5]



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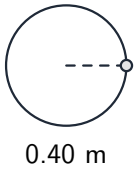


2.1 Convert to frequency.

$$f = \frac{N}{t} = \frac{90}{60\text{ s}} = 1.50\text{ Hz}$$

$$\omega = 2\pi f = 2\pi \times 1.50\text{ s}^{-1} = 9.42\text{ rad s}^{-1}$$

Angular speed. Use radius for linear speed.

$$v = \omega r = 3\pi\text{ s}^{-1} \times 0.40\text{ m} = 3.77\text{ m s}^{-1}$$

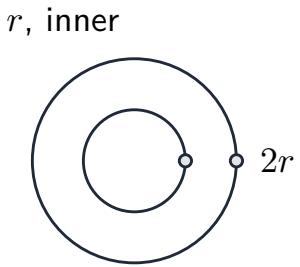
Tangential speed.

$$a = \omega^2 r = (3\pi\text{ s}^{-1})^2 \times 0.40\text{ m} = 35.5\text{ m s}^{-2}$$

Acceleration.

Exercise 2.2

2.2 Two objects rotate on the same rigid disc, at radii r and $2r$. Compare their angular speeds, tangential speeds and accelerations. Explain how an object can accelerate with constant speed. [4]



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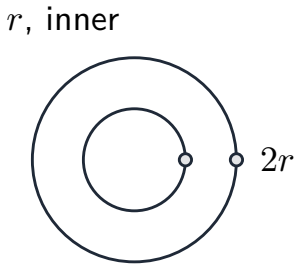
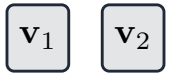


2.2 Angular speeds are equal. From $v = \omega r$, outer tangential speed is twice inner. From $a = \omega^2 r$, outer acceleration is twice inner. Velocity direction changes, so its vector changes despite constant magnitude.

Exercise 2.3

2.3 Use a small-angle velocity-vector triangle to derive $a = v^2/r$. Explain why adding the two velocity vectors does not give the change in velocity. [4]



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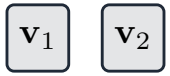


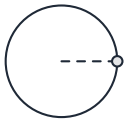
2.3 The change is $\mathbf{v}_2 - \mathbf{v}_1$, drawn between the tips of vectors with a common tail. Similar small-angle triangles give $\Delta v/v \approx \Delta s/r$. With $\Delta s \approx v\Delta t$, divide by time and take the limit:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

The change is inward; addition instead describes a different vector and does not compare final with initial velocity.

Exercise 3.1

3.1 A 0.15 kg piece of clothing follows a vertical drum of radius 0.30 m at constant speed 3.0 m s^{-1}. Calculate the drum's reaction at bottom and top. Explain why these differ even though the required centripetal resultant is equal. State the initial direction of motion if contact and all inward constraint are removed at a point. [6]



$m = 0.15$ kg; $v = 3.0$ m/s

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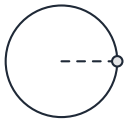


$m = 0.15$ kg; $v = 3.0$ m/s

3.1 Use the resultant of actual forces.

$$F_{N,b} = \frac{mv^2}{r} + mg = \frac{0.15\ \text{kg} \times (3.0\ \text{m s}^{-1})^2}{0.30\ \text{m}} + 0.15\ \text{kg} \times 9.81\ \text{m s}^{-2} = 5.97\ \text{N}$$

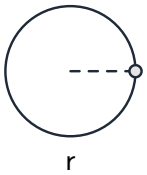
Bottom reaction.

$$F_{N,t} = \frac{mv^2}{r} - mg = \frac{0.15\ \text{kg} \times (3.0\ \text{m s}^{-1})^2}{0.30\ \text{m}} - 0.15\ \text{kg} \times 9.81\ \text{m s}^{-2} = 3.03\ \text{N}$$

Top equation and result. Weight opposes the required inward resultant at the bottom but contributes at the top, so reaction differs. Initial velocity is tangent; gravity can bend the later trajectory.

Exercise 3.2

3.2 A small block rests on a horizontal turntable rotating at 2.0 rad s^{-1}. Maximum friction is $mg/20$. Derive and calculate its largest allowed radius. Explain whether doubling the mass changes this radius under the given model. [5]



$\omega = 2.0$ rad/s

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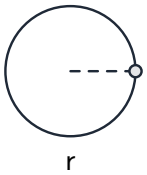


3.2 Friction provides the inward resultant; use its limiting value.

$$mr\omega^2 \le \frac{mg}{20} \quad \Rightarrow \quad r_{\max} = \frac{g}{20\omega^2}$$

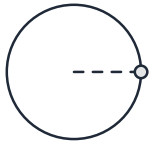
Rearranged limit.

$$r_{\max} = \frac{g}{20\omega^2} = \frac{9.81\ \text{m s}^{-2}}{20(2.0\ \text{s}^{-1})^2} = 0.123\ \text{m}$$

Calculation. Required and limiting forces both scale with mass, so mass cancels and doubling it leaves the radius unchanged.

Exercise 3.3

3.3 A student calls centripetal force “an extra inward force” and says doubling rotational frequency doubles the needed force. Correct both statements using equations. Then determine the minimum speed for a small object to remain in contact at the top of the inside of a vertical circular track of radius 0.50 m. [5]



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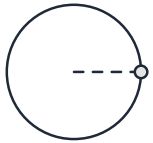

3.3 Centripetal force is the resultant of real forces, not another force. At fixed mass/radius, $F = 4\pi^2 mrf^2$, so doubling frequency quadruples force. At the limiting top speed the reaction is zero.

$$mg = \frac{mv_{\text{min}}^2}{r} \quad \Rightarrow \quad v_{\text{min}} = \sqrt{gr}$$

$$v_{\text{min}} = \sqrt{gr} = \sqrt{9.81\ \text{m s}^{-2} \times 0.50\ \text{m}} = 2.21\ \text{m s}^{-1}$$

Minimum speed.