Mechanics 1

Edexcel IAL M1 (WME01/01); the applied unit pairing the pure core of IAS Mathematics

Mechanics 1: models, motion and forces

- state and use the modelling assumptions of mechanics;
- use vectors for position, velocity and force;
- solve problems with constant acceleration, and read motion graphs;

Mechanics 1: models, motion and forces (2)

- apply Newton's laws, including to connected particles and pulleys;
- use momentum and **impulse**, friction, **equilibrium** and moments.

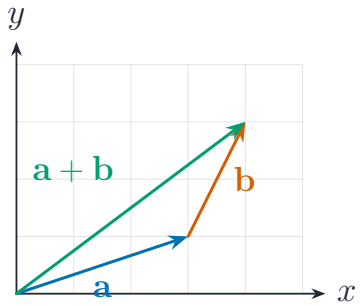
Mathematical models in mechanics

- The words in a question tell you which model to use.
- Questions often ask you to state an assumption, for example "the box is modelled as a particle, so air resistance and rotation are ignored".

Vectors in mechanics

- Displacement, velocity, acceleration and force are vectors.
- With $\mathbf{i}$ pointing east and $\mathbf{j}$ pointing north, the vector $3\mathbf{i} - 4\mathbf{j}$ has magnitude $\sqrt{9+16} = 5$.
- To add vectors, add the $\mathbf{i}$ parts and the $\mathbf{j}$ parts separately.
- The **resultant** 合力 of several forces is their vector sum.

Figure: Vectors in mechanics

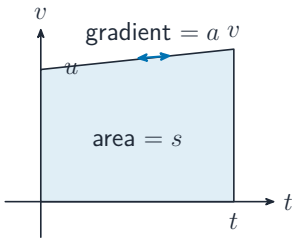


- Adding by the triangle law: draw $\mathbf{b}$ from the tip of $\mathbf{a}$, and $\mathbf{a} + \mathbf{b}$ runs from start to finish.
- Match each part of the diagram to the text.

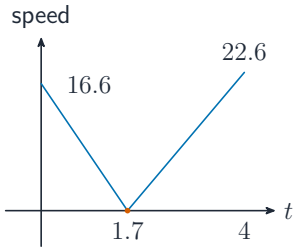
Kinematics of a particle in a straight line

- Take upwards as positive, with A as the origin. Then $s = -12$, $t = 4$, $a = -9.8$.
- $s = ut + \frac{1}{2}at^2$: $-12 = 4u - 4.9 \times 16$, so $4u = 66.4$ and $u = 16.6$ m s^{-1}.
- $v^2 = u^2 + 2as = 16.6^2 + 2(-9.8)(-12) = 510.8$, so the speed is 22.6 m s^{-1}.

Figure: Kinematics of a particle in a straight line



velocity–time: gradient accelerates, area displaces



speed–time V: vertex on the axis, downward leg longer

- Left: a velocity-time trapezium with gradient a and area s marked. Right: the speed-time V of the worked example, vertex on the time axis, labelled 16.6, 22.6 and t = 4.
- Read the axes and units first.

Newton's laws and connected particles

- Perpendicular to the motion, there is no acceleration: $800 \sin 25° = 500 \sin \alpha$, so $\sin \alpha = 0.676$ and $\alpha = 43°$.
- Along the motion: $800 \cos 25° + 500 \cos \alpha - 750 = 15\,000a$, so $a = 0.023$ m s^{-2}.
- For P (moving up): $T - kMg = kM \times \frac{g}{5}$.

Newton's laws and connected particles (2)

- For Q (moving down): $3Mg - T = 3M \times \frac{g}{5}$.
- Add the equations: $3Mg - kMg = \frac{g}{5}(3M + kM)$, so $5(3 - k) = 3 + k$ and $k = 2$.
- If $M = 0.5$ kg, then $T = 1 \times (9.8 + 1.96) = 11.76$ N. The string pulls down on the **pulley** with a force of $2T = 23.5$ N.

Momentum and impulse

- For Q: $3m(v-2u) = 4.5mu$, so $v = 3.5u$.
- For P: the **impulse** is $4.5mu$ backwards, so $km(5u-2u) = 4.5mu$ and $k = 1.5$.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

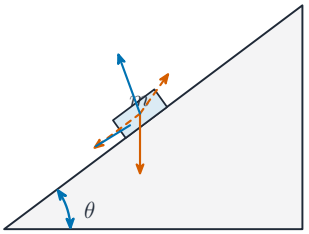
Friction

- If the object is moving, or about to move (in limiting **equilibrium**), $F = \mu R$, where μ is the **coefficient of friction 摩擦系数**.
- If it is at rest and not about to move, $F \leq \mu R$.
- Perpendicular to the slope: $R = mg\cos\theta = \frac{4}{5}mg$.

Friction (2)

- Along the slope (up is positive): friction acts down the slope, so $-mg\sin\theta - \frac{1}{4}R = ma$.
- $a = -g\left(\frac{3}{5} + \frac{1}{5}\right) = -\frac{4}{5}g$, so the deceleration is $\frac{4}{5}g$.

Figure: Friction



- mg (weight)
- $mg\sin\theta$, $mg\cos\theta$
- R (normal reaction)
- $F = \mu R$ (friction)

$\tan\theta = \frac{3}{4}$, so
$\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$

resolve both ways: $R = \frac{4}{5}mg$ and $a = g(\sin\theta + \mu\cos\theta) = \frac{4}{5}g$ when $\mu = \frac{1}{4}$

- A block moving up a rough slope with tan $\theta = 3/4$: the weight resolved into mg sin θ and mg cos θ, the **normal reaction** R, and friction μR down the slope.
- Match each part of the diagram to the text.

Statics of a particle

- weight mg, vertically downwards;
- **normal reaction**, perpendicular to the surface;
- **tension 张力** in a string, pulling away from the particle along the string (a **rod** can also push, which is called thrust);

Statics of a particle (2)

- friction, along the surface.
- Horizontally: $T_1 \sin 40° = T_2 \sin 60°$, so $T_2 = 0.742\, T_1$.
- Vertically: $T_1 \cos 40° + T_2 \cos 60° = 50$, so $T_1(0.766 + 0.371) = 50$.
- $T_1 = 44$ N and $T_2 = 33$ N (2 s.f.).

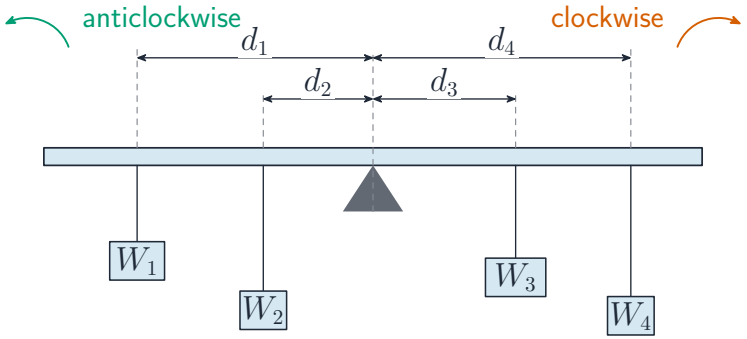
Moments

- the **resultant** force is zero, and
- the total moment about any point is zero.
- About to tilt about A, taking moments about A: $P \times 3a = W \times d$, so $P = \dfrac{Wd}{3a}$.

Moments (2)

- About to tilt about C, taking moments about C: $1.25P \times a = W \times (2a - d)$.
- Substitute: $1.25 \times \dfrac{Wd}{3a} \times a = W(2a - d)$, so $1.25d = 6a - 3d$ and $d = \dfrac{24a}{17}$.

Figure: Moments



- Weights on a rule balanced at a pivot: the clockwise and anticlockwise moments are equal
- Match each part of the diagram to the text.

Sources for this lesson

- Pearson Edexcel, *International Advanced Level Mathematics, Further Mathematics and Pure Mathematics specification* (2018), unit M1.
- G. Galilei, *Discorsi e dimostrazioni matematiche intorno a due nuove scienze* (Leiden, 1638), Third Day.
- Pearson Edexcel, WME01/01 June 2025, questions 1 to 6 and 8 (adapted).

Common errors

- Mixing signs: choose a positive direction for each stage and keep to it.
- Using the constant acceleration equations when the acceleration is not constant.
- Writing one equation for a whole system of connected particles but using the **tension** in it.

Common errors (2)

- Using $F = \mu R$ when the object is at rest and not about to move.
- Using $R = mg$ on a slope instead of $R = mg\cos\theta$.
- Using the distance along a **rod** instead of the perpendicular distance in a moment.
- Giving answers that use $g = 9.8$ to more than 3 significant figures.

Key terms

- **Particle**: a body with mass but no size.
- **Resultant**: the single force equal to the vector sum of all the forces.
- **Impulse**: the change in momentum, $mv - mu$.

Key terms (2)

- **Coefficient of friction**: μ, the ratio of limiting friction to **normal reaction**.
- **Equilibrium**: the state in which the **resultant** force (and the total moment) is zero.
- **Moment**: force multiplied by perpendicular distance from a point.

Check yourself

- 1.
- A car accelerates uniformly from 8 m s^{-1} to 20 m s^{-1} in 6 s.
- Find the acceleration and the distance travelled. 2.
- Find the magnitude and **bearing** of the velocity $6\mathbf{i} + 6\mathbf{j}$ m s^{-1}. 3.