

Pure Mathematics 4

Edexcel IAL P4 (WMA14/01); compulsory for IAL Mathematics and IAL Pure Mathematics

1. Unit P4 at a glance

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WMA14/01: 1 hour 30 minutes, 75 marks; P1–P3 assumed and testable. Must-know: the binomial series $(1+x)^n$ for rational n , volume $V = \pi \int y^2 dx$, and vector notation with $|\mathbf{a}|$ and $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$.

2. Proof by contradiction

Proof by contradiction

Assume the negation, argue to an impossibility, conclude the claim. Classics: $\sqrt{2}$ is irrational (p, q both even contradicts lowest terms); infinitely many primes ($p_1 \cdots p_k + 1$ needs a new prime); no smallest positive rational ($q/2$ beats q).

3. Partial fractions

Partial fractions

$\frac{3x^2+7x+6}{x(x+1)^2} = \frac{6}{x} - \frac{3}{x+1} - \frac{2}{(x+1)^2}$: repeated factors split across powers; improper fractions divide first; verify at a value.

4. Parametric equations

Parametric equations

$x = f(t)$, $y = g(t)$: make t the subject and substitute for the Cartesian form, stating excluded x . Verified paper point: $P(-7, -3)$ at $t = 2$ with gradient $-4/5$ gives normal $5x - 4y + 23 = 0$.

5. Binomial series

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots \text{ for rational } n, |x| < 1.$$

$$\sqrt{4+x} = 2\left(1 + \frac{x}{4}\right)^{1/2} = 2 + \frac{x}{4} - \frac{x^2}{64} + \dots, \text{ valid } |x| < 4.$$

6. Implicit and parametric differentiation

Implicit and parametric differentiation

$\frac{d}{dx}y^2 = 2y\frac{dy}{dx}$; $\frac{d}{dx}(xy) = y + x\frac{dy}{dx}$. Verified example: $2y^2 - 6xy + 7e^{1-2x} = 27$ gives $\frac{dy}{dx} = \frac{6y+14e^{1-2x}}{4y-6x}$, tangent $38x - 13y + 33 = 0$ at $(\frac{1}{2}, 4)$.

7. Differential equations and rates

Differential equations and rates

Separable: $y \frac{dy}{dx} = 3 \sec 2x \tan 2x \Rightarrow \frac{y^2}{2} = \frac{3}{2} \sec 2x + c$; with $y = 4$ at $x = \pi/6$:
 $y^2 = 3 \sec 2x + 10$. Rates: $V = \frac{4}{3} \pi r^3$ with $\frac{dV}{dt} = k$ gives $\frac{dr}{dt} = \frac{k}{4\pi r^2} \propto \frac{1}{r^2}$.

8. Integration methods

Integration methods

Parts: $\int e^x \cos x dx = \frac{1}{2}e^x(\sin x + \cos x) + c$ (the integral returns; collect it).

Substitution converts $\tan x \sec^2 x dx$ with $u = \tan x$ to $\int u du$. Partial fractions integrate to logs.

9. Volumes of revolution

Volumes of revolution

$V = \pi \int y^2 dx$; square first, then identities: $y = 3 \cos x$ on $[0, \frac{\pi}{2}]$ gives

$V = 9\pi \int \cos^2 = \frac{9\pi^2}{4}$. Parametric: $\pi \int y^2 \frac{dx}{dt} dt$.

10. Vectors

Vectors

$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$; unit vector $\mathbf{a}/|\mathbf{a}|$; $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$; angles from $\cos \theta$. Verified: with $\overrightarrow{AB} = (4, 0, 4)$, $\overrightarrow{AC} = (2, \alpha - 2, 8)$ and angle 45° : $\alpha = 2 \pm 4\sqrt{2}$.

Check yourself

Check yourself

Contradiction proofs with the assumption named; partial fractions with repeated factors and improper cases; parametric conversions and gradients; the general binomial series with validity; implicit tangents; separable equations with conditions; parts, substitution and partial-fraction integrals; exact volumes of revolution; vector angles from the scalar product.