

Pure Mathematics 3

Edexcel IAL P3 (WMA13/01); compulsory for IAL Mathematics and IAL Pure Mathematics

1. Unit P3 at a glance

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WMA13/01: 1 hour 30 minutes, 75 marks; P1 and P2 assumed and testable. The must-know list adds compound and double angle formulae, $\sec^2 \equiv 1 + \tan^2$, $\operatorname{cosec}^2 \equiv 1 + \cot^2$, and the derivative and integral tables for $\sin kx$, $\cos kx$, $\tan kx$, e^{kx} , $\ln x$.

2. Functions and inverses

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Domain is the allowed inputs, range the outputs; fg means do g first. Find f^{-1} by writing $y = f(x)$ and solving for x ; verify with $f^{-1}f(x) = x$. Simplify rational expressions by factorising and cancelling factors.

3. Worked: $f(x) = 2x/(3x+1)$

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Range $0 \leq f(x) < \frac{2}{3}$; $f^{-1}(x) = \frac{x}{2-3x}$; solving $f^{-1}(x) = f(x)$ gives $x(9x-3) = 0$, so $x = 0$ or $\frac{1}{3}$.

4. Modulus and combined transformations

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$y = |ax + b|$ is the line with its below-axis half reflected up. Solve by cases and test each root. From $y = f(x)$ sketch $y = |f(x)|$ and $y = f(|x|)$; combine stretches and translations one factor at a time, e.g. $y = 2f(3x)$.

5. Worked: $|2x - 1| = x + 5$

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Case $x \geq \frac{1}{2}$: $2x - 1 = x + 5$, so $x = 6$. Case $x < \frac{1}{2}$: $1 - 2x = x + 5$, so $x = -\frac{4}{3}$. Both pass their case conditions.

6. Reciprocal trig and harmonic form

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$\sec = 1/\cos$, $\operatorname{cosec} = 1/\sin$, $\cot = 1/\tan$, with $\sec^2 \equiv 1 + \tan^2$ and $\operatorname{cosec}^2 \equiv 1 + \cot^2$. Harmonic form: $a \cos \theta + b \sin \theta = R \cos(\theta - \alpha)$, $R = \sqrt{a^2 + b^2}$, $\tan \alpha = b/a$.

7. Worked: $7\cos x - 24\sin x$

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$R = \sqrt{49 + 576} = 25$; $\tan \alpha = \frac{24}{7}$ gives $\alpha = 1.287$:

$7 \cos x - 24 \sin x = 25 \cos(x + 1.287)$, with maximum 25.

8. Log-graph models

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$y = ax^n$: plot $\log y$ vs $\log x$, gradient n , intercept $\log a$. $y = kb^x$: plot $\log y$ vs x , gradient $\log b$, intercept $\log k$. Example: line through $(-3.5, 0)$, $(0, -2)$ gives $y = \frac{1}{100}x^{-4/7}$.

9. Worked: decay model

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$\theta = 21 + 54e^{-0.521t}$: initial 75 °C, floor 21 °C, reaching 25 °C at 5 minutes; the rate $-28.1e^{-0.521t}$ hits -9 °C/min at $T = 2.19$.

10. Differentiation rules

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$\sin kx \rightarrow k \cos kx$, $\cos kx \rightarrow -k \sin kx$, $\tan kx \rightarrow k \sec^2 kx$, $e^{kx} \rightarrow ke^{kx}$, $\ln x \rightarrow \frac{1}{x}$.

Product $(uv)' = u'v + uv'$; quotient $(u/v)' = \frac{u'v - uv'}{v^2}$; chain $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$; implicit flip $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$.

11. Worked: product with a decaying exponential

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$\frac{dy}{dx} = e^{-x^2} (3 \cos 3x - 2x \sin 3x)$; stationary points satisfy $3 \cos 3x = 2x \sin 3x$, i.e.
 $x = \frac{1}{3} \arctan \frac{3}{2x}$.

12. Integration forms

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$\int \sin^2 3x \, dx = \frac{x}{2} - \frac{\sin 6x}{12} + c$ (double angle first). Recognition: $\int \frac{f'}{f} = \ln |f| + c$ and $\int f(f)^n = \frac{(f)^{n+1}}{n+1} + c$.

13. Numerical methods

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Sign change: $f(0.4295) < 0 < f(0.4305)$ with continuity pins a root to 0.430 (3 d.p.).

Iteration $x_{n+1} = \frac{1}{3} \arctan \frac{3}{2x_n}$ from $x_1 = 0.4$: $x_2 = 0.4367$, $x_4 = 0.4307$.

Check yourself

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Rational simplification and inverses verified by composing; modulus cases checked; combined transformations; sec, cosec, cot with identities; harmonic form with exact R; compound and double angle proofs; log-graph recovery; growth-decay models with floor; product, quotient, chain and implicit differentiation; recognition integrals; sign-change and iteration methods.