Pure Mathematics 2

Edexcel IAL P2 (WMA12/01); compulsory for IAS/IAL Mathematics and IAS/IAL Pure Mathematics

1. Unit P2 at a glance

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WMA12/01: 1 hour 30 minutes, 75 marks. P1 knowledge is assumed and may be tested. Calculator and yellow formulae booklet as for P1; the P2 must-know list ($\log_a x + \log_a y \equiv \log_a(xy)$, $\tan\theta \equiv \sin\theta/\cos\theta$, $\sin^2\theta + \cos^2\theta \equiv 1$, area $= \int y\,dx$) is not in the booklet.

2. Proof: exhaustion and counterexample

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A proof proceeds from assumptions through logic to a conclusion. Exhaustion checks every case ($m = 3q + 1$ or $3q + 2$ covers all non-multiples of 3); one counterexample disproves a claim ($2^3 + 1 = 9$ is not prime).

3. Worked: $m^2 + 3m + 2$ divisible by 3

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$(m+1)(m+2)$ with $m = 3q+1$ gives $3(3q+2)(q+1)$; with $m = 3q+2$ gives $3(q+1)(3q+4)$. Both cases are multiples of 3, which exhausts every non-multiple of 3.

4. Factor and remainder theorems

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$(x - b)$ is a factor $\iff f(b) = 0$; dividing by $(ax - b)$ leaves remainder $f(b/a)$. After division, $f(x) = (ax - b)(\text{quotient}) + \text{remainder}$.

5. Worked: two conditions, two unknowns

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$f(x) = ax^3 + bx^2 + 5x - 3$ with $(x+3)$ a factor and remainder 7 dividing by $(2x-1)$: $f(-3) = 0$ gives $b = 3a + 2$; $f(\frac{1}{2}) = 7$ gives $a + 2b = 60$; so $a = 8$, $b = 26$.

6. Circle geometry

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$(x-a)^2+(y-b)^2=r^2$; complete the square on the expanded form. Diameter endpoints give the centre as midpoint and r^2 as a quarter of the diameter length squared. Tangent $\perp$ radius; centre-to-chord perpendicular bisects the chord.

7. Worked: circle from a diameter

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Endpoints $(-2, 5)$ and $(4, 15)$: centre $(1, 10)$, $d^2 = 36 + 100 = 136$ so $r^2 = 34$: $(x-1)^2 + (y-10)^2 = 34$. Nearest point to the x-axis: $(1,\ 10 - \sqrt{34})$.

8. Arithmetic and geometric series

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$a_n = a + (n-1)d$, $S_n = \frac{n}{2}\{2a + (n-1)d\}$; $a_n = ar^{n-1}$, $S_n = \frac{a(1-r^n)}{1-r}$, $S_\infty = \frac{a}{1-r}$ for $|r| < 1$. Both sum-formula proofs should be known; Σ notation is expected.

9. Worked: logs find the term number

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$10(0.8)^{k-1} < 0.0005 \implies (0.8)^{k-1} < 0.00005 \implies (k-1)\log 0.8 < \log 0.00005$, and dividing by the negative $\log 0.8$ flips the sign: $k-1 > 44.38$, so $k = 46$.

10. Binomial expansion

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$(a+bx)^n = a^n + \binom{n}{1}a^{n-1}(bx) + \binom{n}{2}a^{n-2}(bx)^2 + \cdots$ with $\binom{n}{r} = \frac{n!}{r!(n-r)!}$. Match coefficients term by term: in $(2+kx)(1-3x)^5$, the x^2 coefficient $180 - 15k$.

11. Log laws and domains

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$\log_a x + \log_a y \equiv \log_a(xy)$, $\log_a x - \log_a y \equiv \log_a \frac{x}{y}$, $k \log_a x \equiv \log_a(x^k)$. Logs need positive arguments: test every root in the original equation before accepting it.

12. Worked: exact ($\log_3 2$, 12)

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$3^{2x+1} = 6 \times 3^x \implies 3u^2 = 6u$ with $u = 3^x \implies 3^x = 2 \implies x = \log_3 2$, and $y = 6 \times 2 = 12$.

13. Identities into equations

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$\tan\theta \equiv \sin\theta/\cos\theta$ and $\sin^2\theta + \cos^2\theta \equiv 1$ turn mixed equations into one function — often a hidden quadratic. Factor rather than divide: dividing by $\cos\theta$ loses candidates.

14. Worked: $3\tan(2x + \pi/5) = \sqrt{3}$

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$\tan(2x + \frac{\pi}{5}) = \frac{1}{\sqrt{3}} = \tan\frac{\pi}{6}$, so $2x + \frac{\pi}{5} = \frac{\pi}{6} + k\pi$ and $x = \frac{29\pi}{60}$ or $\frac{59\pi}{60}$ in $0 \leq x < \pi$.

15. Stationary points

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$y' = 0$ at stationary points; classify with y'' (positive $=$ minimum, negative $=$ maximum) or the sign change of y'. Increasing where $y' > 0$, decreasing where $y' < 0$. In modelling, eliminate the second variable with the constraint first.

16. Worked: minimum surface

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Trough: $3x^2L = 12$ so $L = 4/x^2$ and $S = 6x^2 + 5xL = 6x^2 + \frac{20}{x}$. Then $S' = 12x - \frac{20}{x^2} = 0 \implies x^3 = \frac{5}{3}$, $S = 25.3$ m^2, and $S'' = 12 + \frac{40}{x^3} > 0$.

17. Definite integrals as areas

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$\int_a^b y\,dx = F(b) - F(a)$ is the area under the curve for $y \geq 0$. Between a curve and a line, sketch first and integrate (upper $-$ lower) between the intersections.

18. Worked: area 13.5 between curve and chord

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$y = 3x + \frac{16}{x^2} - 8$ with $P(1, 11)$, $Q(4, 5)$: line $y = -2x + 13$ gives area 24; the curve gives $\left[\frac{3}{2}x^2 - \frac{16}{x} - 8x\right]_1^4 = 10.5$; region $= 24 - 10.5 = 13.5$.

19. The trapezium rule

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$\int_a^b y\,dx \approx \frac{h}{2}\left[y_0 + y_n + 2(y_1 + \cdots + y_{n-1})\right]$. Convex curve (bends upwards, like x^2): chords sit above it — overestimate. Concave (like $\sqrt{x}$): chords below — underestimate. More strips improve the estimate; scaled integrals scale the answer ($\int \frac{f}{2} = \frac{1}{2}\int f$).

Check yourself

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Exhaustion with complete cases; counterexamples; factor/remainder theorems with $(ax - b)$; circles from any form plus the three properties; both series families with proofs and logs; binomial coefficients; log laws with domain checks; identity-reduced trig equations in either unit; stationary points classified and modelled; areas by definite integral and by the trapezium rule with the error direction.