Solutions

2.1 Pivot (position 2 of 4) = 9: less = 5, 2, 6; greater = none → 5, 2, 6, **9**. Sublist 5, 2, 6 (pivot 2): less none; greater = 5, 6 → **2**, 5, 6. Sublist 5, 6 (pivot 5): less none; greater = 6 → **5**, 6. Final: 2, 5, 6, 9.

2.2 From R: edges RS 4, RT 3 – take RT(3). From {R,T}: edges to new vertices are RS 4, TS 1, TU 2 – take TS(1). From {R,T,S}: edges to U are SU 5 and TU 2 – take TU(2). MST = RT + TS + TU, total 6.

2.3 Vertices: $(0, 0)$: 0; $(5, 0)$ $(2x + y = 10)$: 5; $(2, 6)$ (intersection: $2x + y = 10, x + y = 8$ $\to x = 2, y = 6$): $2 + 12 = 14$; $(0, 8)$: 16. Maximum 16 at $(0, 8)$.

3.1 (a) Total = 4 + 2 + 3 + 5 + 6 + 2 + 4 + 1 = 27. (b) Degrees: P 3 (PQ, PR, PS), Q 3 (PQ, QR, QT), R 4 (QR, RS, PR, RT), S 3 (RS, ST, PS), T 3 (ST, QT, RT): odd villages P, Q, S, T. (c) Pairings of {P, Q, S, T} (weights are shortest paths where no direct edge exists): PQ + ST = 4 + 5 = 9; PS + QT = 2 + 4 = 6; PT + QS = 7 + 5 = 12 (PT via PS–ST or PR–RT, both 7; QS via QR–RS, 5). Least = PS + QT = 6. (d) Route length = 27 + 6 = 33.

3.2 (a) Forward pass: A 0–2, F 0–3 (F after A: starts 2, ends 3), B 0–5, C starts 2 ends 5, D starts 5 ends 9, E starts 9 ends 11. Duration 11; critical path B–D–E (5 + 4 + 2 = 11). (b) F's latest finish = latest start of nothing downstream (F is an end activity): latest finish 11, earliest start 2, duration 1: float = 11 − 2 − 1 = 8 days. (c) Vertices of $\{x \leq 3, y \leq 2, x + y \leq 4\}$: $(0, 0)$, $(3, 0)$, $(2, 2)$, $(0, 2)$: $W = 0, 3, 4, 2$: optimum $W = 4$ at $(2, 2)$ – already integral ✓.

Local review guidance: pairing weights use shortest paths when the direct edge is absent; end activities' latest finish is the project duration; integer optima confirmed at neighbouring lattice points. This is authored classroom guidance.