

Solutions

2.1 Maximise $P = 3x + 5y$ subject to $2x + 4y \leq 200$ (flour), $x + 1.5y \leq 60$ (time), $x, y \geq 0$.

2.2 The region is bounded by $y = 2$ below, $x = 5$ right and $x + y = 8$ above: vertices $(0, 2)$ (y -axis meets $y = 2$), $(0, 8)$ (y -axis meets $x + y = 8$), $(5, 3)$ ($x = 5$ meets $x + y = 8$) and $(5, 2)$ ($x = 5$ meets $y = 2$).

2.3 C at the vertices: $(0, 2)$: 6; $(0, 8)$: 24; $(5, 3)$: 29; $(5, 2)$: 26. Minimum $C = 6$ at $(0, 2)$.

3.1 (a) Maximise $P = 40x + 70y$ subject to $2x + 4y \leq 48$, $x + 2y \leq 20$, $y \leq 6$, $x, y \geq 0$.
(b) The carpentry constraint $2x + 4y \leq 48$ is equivalent to $x + 2y \leq 24$, which never binds alongside $x + 2y \leq 20$ – it is redundant. Vertices and profits: $(0, 0)$: 0; $(10, 0)$ ($x + 2y = 20$): 400; $(8, 6)$ ($x + 2y = 20$, $y = 6$): $40(8) + 70(6) = 740$; $(0, 6)$: 420. Maximum $P = 740$ at $(8, 6)$. (c) $(8, 6)$ is already integral: 8 chairs and 6 tables ✓ (check neighbouring integers $(8, 5)$: $P = 320 + 350 = 670 < 740$; $(9, 5)$: violates $x + 2y \leq 20$? $9 + 10 = 19 \leq 20$ ✓ and $2(9) + 4(5) = 38 \leq 48$ ✓, $P = 360 + 350 = 710 < 740$ ✓).

3.2 (a) The region lies above both lines; vertices: $(0, 10)$ (on $2x + y = 10$), $(4, 2)$ (intersection of $x + 2y = 8$ and $2x + y = 10$: solving, $y = 2$, $x = 4$), and $(8, 0)$ (on $x + 2y = 8$); the region extends to infinity up-right. (b) C at the vertices: $(0, 10)$: 40; $(4, 2)$: 28; $(8, 0)$: 40. Minimum $C = 28$ at $(4, 2)$. (c) Increasing x or y only adds cost (coefficients positive), so the unbounded directions cannot lower C ; the minimum must occur at a vertex.

Local review guidance: every constraint captured with non-negativity; vertices found as constraint intersections, tested in the objective; integer checks at neighbouring lattice points inside the region. This is authored classroom guidance.