Solutions

2.1 Events: start s; A: s→e1 (2), B: s→e2 (5); C: e1→e3 (3); D: e1→e3 and e2→e3 – D depends on both A and B, so e1 must connect to e3 and e2 to e3 (a dummy from e2 to e1 if drawn with D leaving a shared node). Durations: A–C = 2 + 3 = 5; B = 5; D needs both done: D's earliest start = max(2, 5) = 5. Project duration = max(5, 5 + 1) = 6 days.

2.2 Earliest = latest at every event: every activity on the chain is critical, float 0.

2.3 Float = 16 − 5 − 6 = 5 days.

3.1 (a) Arcs: A from start to event 2; B from start to event 3; C from event 2 to event 4; D depends on A and B, so a dummy (0) runs from event 2 to event 3 and D leaves event 3 – but D must end at its OWN event 5 (time 4 + 2 = 6), since F follows D alone and must not inherit C's dependency. A dummy from event 5 to event 4 carries D into E's start (event 4 = max(C's 10, D's 6) = 10); E runs from event 4 to event 6; F runs from event 5 to event 7; a dummy from event 6 to event 7 gives the single finish. (b) Forward pass: event 1 = 0; event 2 = 4 (A); event 3 = max(3, 4 + 0) = 4; event 5 = 4 + 2 = 6 (D); event 4 = max(4 + 6, 6 + 0) = 10; event 6 = 10 + 5 = 15; event 7 = max(15 + 0, 6 + 3) = 15. Backward pass: event 7 = 15; event 6 = 15; event 4 = 10; event 5 = min(latest of event 4 − 0, latest of event 7 − 3) = min(10, 12) = 10; event 3 = 10 − 2 = 8; event 2 = min(8 − 0, 10 − 6) = 4; event 1 = min(4 − 4, 8 − 3) = 0 ✓. (c) Critical events: 1 (0/0), 2 (4/4), 4 (10/10), 6 (15/15), 7 (15/15): critical path A–C–E, duration 15 days. (d) F's earliest start is event 5's earliest time 6, its latest finish 15: float = 15 − 6 − 3 = 6 days.

3.2 (a) Bars: A from 0 for 4; B from 0 for 3; C from 4 for 6; D from 4 for 2; E from 10 for 5; F from 6 for 3 with its slack after it. Critical activities A, C, E on one unbroken chain. (b) One worker suffices for the critical chain if nothing else overlaps it in time – but B (0–3) and D (4–6) overlap A (0–4) and C (4–10): at least 2 workers are needed. (c) F's latest finish is 15 and it lasts 3 days, so it can start as late as 15 − 3 = 12 (its earliest start is 6, giving the 6-day float).

Local review guidance: dummies carry exactly the dependency that shared endpoints cannot; forward pass takes maxima, backward pass minima; float from latest finish − earliest start − duration; Gantt bars from earliest starts with slack trailing. This is authored classroom guidance.

3.3

(a)

- forward pass, the largest of the incoming totals: $e_1 = 0$, $e_2 = 3$, $e_3 = \max(5,\ 3{+}2) = 5$, $e_4 = 3 + 4 = 7$, $e_5 = \max(5 + 6,\ 7 + 3) = 11$, $e_6 = \max(7 + 4,\ 11 + 2) = 13$
- backward pass, the smallest of the outgoing differences: $l_6 = 13$, $l_5 = 11$, $l_4 = \min(11 - 3,\ 13 - 4) = 8$, $l_3 = 5$, $l_2 = \min(8 - 4,\ 5 - 2) = 3$, $l_1 = 0$

(b)

- events 1, 2, 3, 5, 6 have $e = l$, so the zero-float activities are **A, B, D, E, H** (paths A–D–E–H and B–E–H); project length **13 days**

(c)

- total float = late time of the end event − early time of the start event − duration:

  $\text{float}(C) = 8 - 3 - 4 = \mathbf{1}$, $\text{float}(F) = 11 - 7 - 3 = \mathbf{1}$, $\text{float}(G) = 13 - 7 - 4 = \mathbf{2}$

(d)

- lower bound = total activity time ÷ project length = $\dfrac{29}{13} = 2.23$, rounded up to **3** workers