Solutions

2.1 Degrees: A 3 (AB, AD, AC), B 2, C 3 (BC, CD, CA), D 2 – odd nodes A and C. The only pairing is AC (6): route length 20 + 6 = 26.

2.2 From W: nearest X (3); from X: nearest Y (2); from Y: nearest Z (7)? YZ = 7 vs YW = 4 but W is visited – from Y the unvisited are Z only: Y–Z (7); return Z–W (5). Tour W–X–Y–Z–W = 3 + 2 + 7 + 5 = 17.

2.3 Doubled MST: 24. A closed walk along each MST edge twice visits every vertex but repeats a vertex whenever the tree branches; replacing such repeated stretches by direct edges can only shorten the tour because the triangle inequality makes the direct edge no longer than the two-leg path.

3.1 (a) Degrees: A 3 (AB, AC, AD), B 3 (AB, BC, BD), C 3 (AC, BC, CD), D 3 (CD, BD, AD) – all four vertices are odd. (b) Pairings: AB + CD = 6 + 2 = 8; AC + BD = 4 + 7 = 11; AD + BC = 3 + 5 = 8. Least = 8 (either AB + CD or AD + BC). (c) Total of all edges = 6 + 4 + 5 + 2 + 7 + 3 = 27; shortest route = 27 + 8 = 35. (d) Yes: four odd-degree vertices cannot form a closed trail using each edge once – exactly the chosen pairing's edges (AB and CD, or AD and BC) must be repeated.

3.2 (a) From J: nearest L (5); from L: nearest K (4); from K: nearest M (10); return M–J (9). Tour J–L–K–M–J = 5 + 4 + 10 + 9 = 28. (b) MST (Kruskal): KL 4 ✓, JL 5 ✓, LM 6 ✓ (tree on J, K, L, M), total 15. Deleting J: MST of {K, L, M} = KL 4 + LM 6 = 10, plus J's two cheapest edges JL 5 + JK 8 = 13: lower bound = 10 + 13 = 23. (c) The optimal tour lies between 23 and 28 (the nearest-neighbour upper bound).

Local review guidance: odd nodes listed with degrees; ALL pairings costed before choosing; postman total = edges + chosen pairing; NN tour and the delete-a-vertex lower bound assembled from the MST of the rest plus the two cheapest at the deleted vertex. This is authored classroom guidance.