9.2 Minimum spanning trees and shortest paths

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS minimum spanning tree • Dijkstra's algorithm

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Objective

Find minimum spanning trees by Kruskal and Prim (node and matrix forms) and run Dijkstra with labels in order of permanence, recovering routes (objectives 2.1–2.2).

1 Key knowledge and worked examples

MST. Kruskal: sort all edges, add the smallest that does not form a cycle (reject others); Prim: from a start vertex repeatedly add the cheapest edge joining the tree to a new vertex; the matrix form crosses out rows and rings cheapest column entries. Dijkstra: permanent labels smallest-first; update neighbours (new = permanent + weight if smaller); recover the route by tracing back.

Worked example (original). Edges AB 4, AC 2, BC 1, BD 5, CD 8. Kruskal: BC 1 ✓, AC 2 ✓ (the tree holds A, B, C), AB 4 forms a cycle (rejected), BD 5 adds D ✓, CD 8 rejected (cycle): MST = BC + AC + BD, total 8. Prim from A: AC 2, CB 1, BD 5 – the same tree. Dijkstra A to D: A 0, C 2, B 3, D 8; route A–C–B–D.

2 Practice

2.1 Edges AB 3, AC 1, BC 2, CD 4, AD 5: find the MST by Kruskal, listing edges in order of choice, and give its total weight. [3]

2.2 For the same network, run Prim from vertex B and show the MST is the same. [3]

2.3 Edges AB 2, BC 3, CD 1, AD 4, AC 2: use Dijkstra from A to find the shortest distance to D, showing each label as it becomes permanent, and state the route. [4]

3 Exam-style questions

3.1 *(Original.)* A network connects four villages P, Q, R, S with edge weights PQ 7, PR 3, PS 8, QR 2, RS 6, QS 5 (km). (a) Use Kruskal's algorithm to find the minimum spanning tree, listing the edges in the order chosen and rejecting cycles explicitly. (b) Use Prim's algorithm from P to find the same tree. (c) State the total weight of the MST and explain why both algorithms must agree. [8]

3.2 *(Original.)* With the same network, use Dijkstra's algorithm from P to find the shortest route from P to S, showing the order in which labels become permanent and the final route with its length. [6]