Solutions

2.1 Mean 7; variance $\frac{64}{12} = \frac{16}{3}$; $P(5 < X < 9) = \frac{4}{8} = \frac{1}{2}$.

2.2 $\frac{(b-a)^2}{12} = \frac{64}{3}$ gives $b - a = 16$; $a + b = 20$; so $a = 2$, $b = 18$.

2.3 $\int_0^4 \frac{x}{c} = \frac{8}{c} = 1$ gives $c = 8$; $E(X) = \frac{1}{8}\int_0^4 x^2 = \frac{64}{24} = \frac{8}{3}$.

3.1 (a) Normalising: $\int_0^k \frac{2x}{21}dx + \int_k^6 \frac{2(6-x)}{15}dx = 1$, i.e. $\frac{k^2}{21} + \frac{2}{15}\left[6x - \frac{x^2}{2}\right]_k^6 = \frac{k^2}{21} + \frac{2}{15}(18 - 6k + \frac{k^2}{2}) = 1$. Multiplying by 105: $5k^2 + 14(18 - 6k + \frac{k^2}{2}) = 105$, so $5k^2 + 252 - 84k + 7k^2 = 105$: $12k^2 - 84k + 147 = 0$, i.e. $4k^2 - 28k + 49 = 0 - (2k - 7)^2 = 0$, so $k = 3.5$ ✓. (b) $E(X) = \frac{2}{21}\int_0^{3.5} x^2 dx + \frac{2}{15}\int_{3.5}^6 (6x - x^2)dx$. The first: $\frac{2}{21}\left[\frac{x^3}{3}\right]_0^{3.5} = \frac{2}{21} \cdot \frac{343}{24} = \frac{49}{36}$ (exactly $\frac{343}{252}$). The second: $\frac{2}{15}\left[3x^2 - \frac{x^3}{3}\right]_{3.5}^6$ with $3(3.5)^2 - \frac{(3.5)^3}{3} = \frac{147}{4} - \frac{343}{24} = \frac{539}{24}$, so the bracket is $36 - \frac{539}{24} = \frac{325}{24}$ and the term is $\frac{650}{360} = \frac{65}{36}$. Total: $E(X) = \frac{49+65}{36} = \frac{114}{36} = \frac{19}{6}$ ✓ (awrt 3.17). (c) $\text{Var} = \frac{277}{24} - (\frac{19}{6})^2 = \frac{277}{24} - \frac{361}{36} = \frac{831-722}{72} = \frac{109}{72}$ (awrt 1.51).

3.2 (a) $\frac{3}{b-a} = \frac{3}{16}$ gives $b - a = 16$; $\frac{a+b}{2} = 6$ gives $a + b = 12$: $2b = 28$, so $b = 14$, $a = -2$ (accept $a = 4.4$, $b = 7.6$). (b) $E(cX+1) = 6c+1 = 3$ gives $c = \frac{1}{3}$. (c) $\text{Var}(X) = \frac{16^2}{12} = \frac{64}{3}$. (d) $E(X^2) = \frac{64}{3} + 36 = \frac{172}{3}$. (e) $\frac{3}{2}X - 14 > -2$ gives $X > 8$: $P(X > 8) = \frac{14-8}{16} = \frac{3}{8}$.

Local review guidance: normalisation with both integrals over their own limits and the exact factorisation; moments by integration before any subtraction; uniform answers as exact fractions with the chosen pair of roots used consistently. This is authored classroom guidance.