

Solutions

2.1 Magnitude $\sqrt{9+16} = 5 \text{ m s}^{-1}$. The direction $(3, -4)$ is south-east; $\tan^{-1}(3/4) = 36.9^\circ$ east of south, so the bearing is $180 - 37 = 143^\circ$.

2.2 Resultant $6\mathbf{i} - 2\mathbf{j}$: magnitude $\sqrt{36+4} = 2\sqrt{10} \approx 6.3 \text{ N}$, direction $\tan^{-1}(2/6) = 18.4^\circ$ south of east.

2.3 Velocity $= \frac{(12\mathbf{i} + 7\mathbf{j}) - (3\mathbf{i} - 2\mathbf{j})}{3} = 3\mathbf{i} + 3\mathbf{j}$; $\mathbf{r}(t) = (3\mathbf{i} - 2\mathbf{j}) + (t - 1)(3\mathbf{i} + 3\mathbf{j}) = 3t\mathbf{i} + (3t - 5)\mathbf{j}$.

2.4 Perpendicular $\iff$ dot product zero: $3\lambda - 8 = 0$, so $\lambda = \frac{8}{3}$.

3.1 (a) $\frac{(10\mathbf{i} + 4\mathbf{j}) - (-5\mathbf{i} + 16\mathbf{j})}{3} = \frac{15\mathbf{i} - 12\mathbf{j}}{3} = (5\mathbf{i} - 4\mathbf{j}) \text{ m s}^{-1}$. (b) $\mathbf{r} = (-5\mathbf{i} + 16\mathbf{j}) + (t - 2)(5\mathbf{i} - 4\mathbf{j}) = (5t - 15)\mathbf{i} + (24 - 4t)\mathbf{j}$. (c) At $t = 5$: A is at $10\mathbf{i} + 4\mathbf{j}$; B is at $-5\mathbf{i} + 7\mathbf{j} + 5(2\mathbf{i} - 3\mathbf{j}) = 5\mathbf{i} - 8\mathbf{j}$; so $\overrightarrow{AB} = -5\mathbf{i} - 12\mathbf{j}$, magnitude 13, direction $\tan^{-1}(5/12) = 22.6^\circ$ west of due south: bearing 203° .

3.2 (a) Position $(2, -1)$: south-east quadrant, $\tan^{-1}(2/1) = 63.4^\circ$ east of south, bearing $180 - 63 = 117^\circ$. (b) Position $(2 + 3t)\mathbf{i} + (-1 + 4t)\mathbf{j}$: due east when the $\mathbf{j}$ -component is zero, $t = 0.25 \text{ h}$; position $2.75\mathbf{i}$, so the ship is 2.75 km due east of the origin.

Local review guidance: difference quotient with the correct time interval; position vectors generalised from one instant; bearings measured clockwise from north in the correct quadrant. This is authored classroom guidance.