Solutions

2.1 Magnitude 5; unit vector $\frac{1}{5}(3\mathbf{i}+4\mathbf{j})$.

2.2 $\overrightarrow{PQ} = 3\mathbf{i}+4\mathbf{j}$, magnitude 5.

2.3 $\mathbf{b} = -2\mathbf{a}$: yes, parallel (one is a scalar multiple of the other).

2.4 Dot product $= 0+0+1 = 1$; magnitudes $\sqrt{2}$ each:

$$\cos\theta = \frac{1}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2} \implies \theta = 60°$$

3.1 $\overrightarrow{AB} = 2\mathbf{i}+2\mathbf{j}+4\mathbf{k}$, $|AB| = \sqrt{24}$; $\overrightarrow{AC} = 1\mathbf{i}+(\beta+1)\mathbf{j}+2\mathbf{k}$; dot product $= 2+2(\beta+1)+8 = 2\beta+12$.

$$\cos 60° = \frac{2\beta+12}{\sqrt{24}\sqrt{1+(\beta+1)^2+4}} = \frac{1}{2}$$

$$2(2\beta+12) = \sqrt{24}\sqrt{(\beta+1)^2+5} \implies 16(\beta+6)^2 = 24((\beta+1)^2+5)$$

$$2(\beta+6)^2 = 3((\beta+1)^2+5) \implies 2\beta^2+24\beta+72 = 3\beta^2+6\beta+18$$

$$\beta^2-18\beta-54 = 0 \implies \beta = \frac{18\pm\sqrt{540}}{2} = 9\pm 3\sqrt{15}$$

- Check $\beta = 9+3\sqrt{15} \approx 20.62$: dot $= 53.2$; $|AC| = \sqrt{1+467.4+4} = 21.7$; $\cos\theta = 53.2/(4.90\times 21.7) = 0.500 = \frac{1}{2}$ ✓ exactly.

3.2 $\overrightarrow{OR} = 3\mathbf{i}+(-1)\mathbf{j}+4\mathbf{k}$; $\overrightarrow{OM} = \frac{1}{2}(\overrightarrow{OR}+\overrightarrow{OS}) \Rightarrow \overrightarrow{OS} = 2\overrightarrow{OM}-\overrightarrow{OR} = (-1)\mathbf{i}+3\mathbf{j}+(-4)\mathbf{k}$.

$$\overrightarrow{RS} = \overrightarrow{OS}-\overrightarrow{OR} = (-4)\mathbf{i}+4\mathbf{j}+(-8)\mathbf{k}, \qquad RS = \sqrt{16+16+64} = \sqrt{96} = 4\sqrt{6}$$

- Check: M is the midpoint of R and S: $\frac{1}{2}(3+(-1)) = 1$, $\frac{1}{2}(-1+3) = 1$, $\frac{1}{2}(4+(-4)) = 0$ ✓.

Local review guidance: correct $\overrightarrow{AB}$, $\overrightarrow{AC}$ and scalar product; exact β or distance; angle equation set up with magnitudes. This is authored classroom guidance.

3.3

(a)

- $F(4,3,2)$, $A(4,0,0)$, $G(0,3,2)$, so $\overrightarrow{OF} = 4\mathbf{i}+3\mathbf{j}+2\mathbf{k}$ and $\overrightarrow{AG} = -4\mathbf{i}+3\mathbf{j}+2\mathbf{k}$

(b)

- the angle between two vectors, $\cos\theta = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$:

$$\cos\theta = \frac{4(-4)+3(3)+2(2)}{\sqrt{29}\sqrt{29}} = -\frac{3}{29} \Rightarrow \theta = 95.9°$$

- the acute angle between the lines is $180° - 95.9° = \mathbf{84.1°}$

(c)

- X has position vector $\lambda(4\mathbf{i}+3\mathbf{j}+2\mathbf{k})$, and $\overrightarrow{BX}\cdot\overrightarrow{OF} = 0$ with $B(4,3,0)$:

$$(4\lambda-4)4+(3\lambda-3)3+(2\lambda)2 = 0 \Rightarrow 29\lambda = 25 \Rightarrow \lambda = \tfrac{25}{29}$$

- $\overrightarrow{OX} = \frac{25}{29}(4\mathbf{i}+3\mathbf{j}+2\mathbf{k})$, so $\overrightarrow{BX} = \frac{1}{29}(-16\mathbf{i}-12\mathbf{j}+50\mathbf{k})$
- $|BX| = \sqrt{x^2+y^2+z^2}$:

$$|BX| = \frac{1}{29}\sqrt{16^2+12^2+50^2} = \frac{\sqrt{2900}}{29} = \frac{\mathbf{10}}{\sqrt{\mathbf{29}}}$$