

Solutions

2.1

$$1 + \frac{x}{3} - \frac{x^2}{9} + \dots$$

Coefficient of x^2 : $\frac{1}{3}(-\frac{2}{3}) \cdot \frac{1}{2} = -\frac{1}{9}$.

2.2

$$(2-x)^{-1} = \frac{1}{2} \left(1 - \frac{x}{2}\right)^{-1} = \frac{1}{2} + \frac{x}{4} + \frac{x^2}{8}, \quad |x| < 2.$$

2.3

$$\sqrt{9+x} = 3 \left(1 + \frac{x}{9}\right)^{1/2} = 3 + \frac{x}{6} - \frac{x^2}{216} + \dots$$

2.4 $x = 0.4$: $3 + 0.0667 - 0.00074 = 3.0659$; true $\sqrt{9.4} = 3.06594$ ✓.

3.1 (a) From the worked example: $2 + \frac{x}{4} - \frac{x^2}{64}$. (b) Replace x by $-x$: $2 - \frac{x}{4} - \frac{x^2}{64}$. (c) Divide (b)'s expansion into (a)'s:

$$\frac{2 + \frac{x}{4} - \frac{x^2}{64} + \dots}{2 - \frac{x}{4} - \frac{x^2}{64} + \dots} = \frac{1 + \frac{x}{8} - \frac{x^2}{128}}{1 - \frac{x}{8} - \frac{x^2}{128}} \approx \left(1 + \frac{x}{8}\right) \left(1 + \frac{x}{8}\right) \approx 1 + \frac{x}{4}$$

(to first order; the x^2 terms cancel to this accuracy: $\frac{x}{8} + \frac{x}{8} = \frac{x}{4}$ and $\frac{x^2}{64} - \frac{x^2}{64} - \frac{x^2}{64} = -\frac{x^2}{64}$, an order-2 remainder). With $x = 1$: $\sqrt{\frac{5}{3}} \approx 1.25$; true value 1.2909... – a first-order approximation with visible error, improving with smaller x ✓.

3.2 $\frac{7-x}{(2-x)(1+x)} = \frac{A}{2-x} + \frac{B}{1+x}$: $7-x = A(1+x) + B(2-x)$. At $x = 2$: $5 = 3A \Rightarrow A = \frac{5}{3}$. At $x = -1$: $8 = 3B \Rightarrow B = \frac{8}{3}$.

$$\begin{aligned} \frac{5}{3}(2-x)^{-1} + \frac{8}{3}(1+x)^{-1} &= \frac{5}{3} \left(1 - \frac{x}{2}\right)^{-1} + \frac{8}{3}(1+x)^{-1} \\ &= \frac{5}{3} \left(1 + \frac{x}{2} + \frac{x^2}{4} + \dots\right) + \frac{8}{3}(1-x+x^2-\dots) = \frac{7}{2} - \frac{9}{4}x + \frac{23}{8}x^2 + \dots \end{aligned}$$

valid for $|x| < 1$ (the tighter of $|x| < 2$ and $|x| < 1$). Check at $x = 0$: LHS = $\frac{7}{2}$; series = $\frac{5}{6} + \frac{8}{3} = \frac{7}{2}$ ✓.

Local review guidance: correct n and $\frac{b}{a}$ scaling; simplified fractional coefficients; validity stated and approximations used within it. This is authored classroom guidance.