

4.1 Proof by contradiction and partial fractions

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

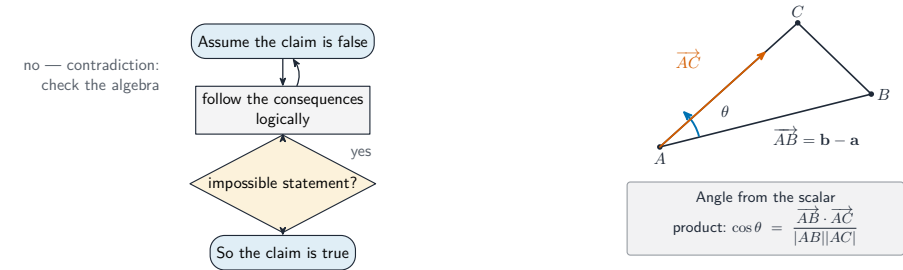
KEY WORDS proof by contradiction • partial fractions • repeated factor
• improper fraction

Objective

Prove statements by contradiction ($\sqrt{2}$ irrational, infinitely many primes, unfamiliar claims) and decompose rational functions into partial fractions, including repeated linear factors and improper fractions. Original teaching questions for specification items P4.1.1 and P4.2.1; the exam-style part adapts reviewed WMA14 June 2025 demand.

1 Key knowledge and worked examples

Proof by contradiction : assume the negation of the claim, argue logically until an impossibility, conclude the claim. **Partial fractions** split a rational function into sums over its denominator’s factors; a repeated factor $(ax + b)^2$ contributes $\frac{B}{ax + b} + \frac{C}{(ax + b)^2}$; improper fractions divide first.



$\sqrt{2}$ irrational and the infinity of primes both run this loop

Name the assumption and the contradiction explicitly.

Worked example – contradiction. Prove there is no smallest positive rational number.

- Suppose q IS the smallest positive rational. Then $\frac{q}{2}$ is positive, rational, and $\frac{q}{2} < q$

– a smaller one exists, contradicting the assumption. ■

Worked example – repeated factor. Decompose $\frac{3x^2 + 7x + 6}{x(x + 1)^2}$.

$$\frac{3x^2 + 7x + 6}{x(x + 1)^2} = \frac{A}{x} + \frac{B}{x + 1} + \frac{C}{(x + 1)^2} \implies 3x^2 + 7x + 6 = A(x + 1)^2 + Bx(x + 1) + Cx$$

- $x = 0$: $A = 6$. $x = -1$: $2 = -C \Rightarrow C = -2$. x^2 coefficients: $3 = A + B \Rightarrow B = -3$.

$$\frac{6}{x} - \frac{3}{x + 1} - \frac{2}{(x + 1)^2}$$

- Check at $x = 1$: LHS = $16/4 = 4$; RHS = $6 - \frac{3}{2} - \frac{1}{2} = 4 \checkmark$.

2 Practice

2.1 Prove by contradiction that $\sqrt{3}$ is irrational. [4]

2.2 Decompose $\frac{7x + 10}{(x + 1)(x + 3)}$. [3]

2.3 Decompose $\frac{5x + 7}{(x - 1)^2}$. [3]

2.4 Write $\frac{x^2 + 2x + 5}{x^2 + x}$ in the form $1 + \frac{A}{x} + \frac{B}{x + 1}$. [4]

3 Exam-style questions

3.1 (*Adapted from Edexcel IAL WMA14 June 2025 Q10.*) Use proof by contradiction to show that $|\sin x + \cos x| < 1$ for all angles x with $90^\circ < x < 180^\circ$. [4]

3.2 Decompose $\frac{2x^2 + 5x + 4}{(x + 2)(x + 1)^2}$ and verify your decomposition at $x = 1$. [5]
