Solutions

2.1 Suppose $\sqrt{3} = \frac{p}{q}$ in lowest terms. Then $3q^2 = p^2$, so p^2 is divisible by 3, so p is (if $p = 3m$, $p^2 = 9m^2$; conversely a prime dividing p^2 divides p). Write $p = 3m$: $3q^2 = 9m^2 \Rightarrow q^2 = 3m^2$, so q is divisible by 3 too – both divisible contradicts lowest terms. ■

2.2 $7x + 10 = A(x + 3) + B(x + 1)$: at $x = -1$, $A = \frac{3}{2}$; at $x = -3$, $B = \frac{11}{2}$. (Or compare coefficients: $A + B = 7$, $3A + B = 10$.)

2.3 With $\dfrac{5x+7}{(x-1)^2} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2}$, multiply through: $5x + 7 = A(x - 1) + B$. Comparing x-coefficients, $A = 5$; comparing constants, $7 = -5 + B \Rightarrow B = 12$.

2.4 $\dfrac{x^2+2x+5}{x^2+x} = 1 + \dfrac{x+5}{x^2+x} = 1 + \dfrac{x+5}{x(x+1)}$; decompose $\dfrac{x+5}{x(x+1)} = \dfrac{5}{x} - \dfrac{4}{x+1}$. So $A = 5$, $B = -4$. Check at $x = 1$: $1 + 5 - 2 = 4$; original $\frac{8}{2} = 4$ ✓.

3.1 Suppose some x in $90° < x < 180°$ has $|\sin x + \cos x| \geq 1$. Squaring: $1 + \sin 2x \geq 1$, so $\sin 2x \geq 0$. But $180° < 2x < 360°$ forces $\sin 2x < 0$ – a contradiction. So $|\sin x + \cos x| < 1$ throughout. ■ Check at $x = 135°$: $|0| = 0 < 1$ ✓.

3.2 $\dfrac{2x^2+5x+4}{(x+2)(x+1)^2} = \dfrac{A}{x+2} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2}$: $2x^2 + 5x + 4 = A(x+1)^2 + B(x+2)(x+1) + C(x+2)$.

- $x = -2$: $8 - 10 + 4 = 2 = A$. $x = -1$: $2 - 5 + 4 = 1 = C$. x^2: $2 = A + B \Rightarrow B = 0$.

$$\frac{2}{x+2} + \frac{1}{(x+1)^2}$$

  Check at $x = 1$: LHS $= \frac{2+5+4}{(1+2)(1+1)^2} = \frac{11}{12}$; RHS $= \frac{2}{3} + \frac{1}{4} = \frac{11}{12}$ ✓.

Local review guidance: assumption of the negation stated, or correct partial-fraction identity set up; exact values; verification or the named contradiction. This is authored classroom guidance.