

# 3.6 Integration and numerical methods

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_ **Total: 26 marks**

**KEY WORDS** recognition forms • sign change • iteration

## Objective

Integrate  $e^{kx}$ ,  $\frac{1}{x}$  and  $\sin / \cos kx$ ; recognise  $\int \frac{f'(x)}{f(x)} dx$  and  $\int f'(x)(f(x))^n dx$ ; use identities on squared trig functions; locate roots by sign change; and run iterations  $x_{n+1} = f(x_n)$ . Original teaching questions for specification items P3.5.1, P3.5.2, P3.6.1 and P3.6.2; the exam-style part adapts reviewed WMA13 June 2025 demand.

## 1 Key knowledge and worked examples

$$\int e^{kx} dx = \frac{1}{k} e^{kx} + c \qquad \int \frac{1}{x} dx = \ln |x| + c$$

$$\int \cos kx \, dx = \frac{1}{k} \sin kx + c \qquad \int \sin kx \, dx = -\frac{1}{k} \cos kx + c$$

**Recognition forms** :

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c \qquad \int f'(x)(f(x))^n dx = \frac{(f(x))^{n+1}}{n+1} + c \quad (n \neq -1)$$

For squares:  $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$ ,  $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$ ,  $\tan^2 A = \sec^2 A - 1$ .

**Numerical methods** : if  $f$  is continuous and  $f(a)f(b) < 0$ , a root lies in  $(a, b)$ ; an iteration  $x_{n+1} = f(x_n)$  from a start value approaches a fixed point when it converges – confirm the final accuracy with a sign change.

**Worked example – identity first** (adapted from Edexcel IAL WMA13 June 2025 Q5). Find  $\int \sin^2 3x \, dx$ .

- $\cos 6x \equiv 1 - 2 \sin^2 3x$ , so  $\sin^2 3x = \frac{1}{2}(1 - \cos 6x)$ :

$$\int \sin^2 3x \, dx = \frac{1}{2} \int (1 - \cos 6x) \, dx = \frac{x}{2} - \frac{\sin 6x}{12} + c$$

- Check: differentiating gives  $\frac{1}{2} - \frac{1}{2} \cos 6x = \sin^2 3x \checkmark$ .

**Worked example – recognition.** Find  $\int \frac{4x}{x^2+3} dx$  and  $\int 2x\sqrt{x^2+3} \, dx$ .

- First: numerator  $4x$  is the derivative of  $x^2 + 3$ :

$$\int \frac{4x}{x^2+3} \, dx = \ln|x^2+3| + c$$

- Second: with  $u = x^2 + 3$ ,  $du = 2x \, dx$ :

$$\int 2x u^{1/2} \, dx = \int u^{1/2} \, du = \frac{2}{3} u^{3/2} + c = \frac{2}{3} (x^2 + 3)^{3/2} + c$$

- Check the second:  $\frac{d}{dx} \frac{2}{3} (x^2 + 3)^{3/2} = \frac{2}{3} \cdot \frac{3}{2} (x^2 + 3)^{1/2} \cdot 2x = 2x\sqrt{x^2+3} \checkmark$ .

## 2 Practice

**2.1** Find  $\int e^{5x} \, dx$ . [2]

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**2.2** Find  $\int \frac{3}{x} \, dx \, (x > 0)$ . [2]

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**2.3** Find  $\int \cos 4x \, dx$ . [2]

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**2.4** Find  $\int \frac{2x+2}{x^2+2x+5} \, dx$ . [3]

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**2.5** Find  $\int 3x^2(x^3 + 1)^4 dx$ . [3]

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**2.6** Show that  $x^3 - 3x + 1 = 0$  has a root between 0 and 1. [2]

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**3 Exam-style questions**

**3.1** (*Adapted from Edexcel IAL WMA13 June 2025 Q5 and Q7.*)

(a) Find  $\int \cos^2 2x dx$ .

(b) Find  $\int \frac{x}{x^2 + 4} dx$ . [5]

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**3.2** (*Adapted from Edexcel IAL WMA13 June 2025 Q7.*) The equation  $x = e^{-x}$  has exactly one positive root.

- (a) Show the root lies between 0.5 and 0.6.
- (b) Use the iteration  $x_{n+1} = e^{-x_n}$  from  $x_1 = 0.5$  to find  $x_3$  to 3 decimal places.
- (c) Verify the root is 0.567 to 3 decimal places. [7]

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