3.5 Differentiation rules and implicit work

Name: _____ Class: _____ Date: _____ **Total: 22 marks**

KEY WORDS product rule • quotient rule • chain rule • implicit differentiation

Objective

Differentiate $\sin kx$, $\cos kx$, $\tan kx$, e^{kx} and $\ln x$; apply the product, quotient and chain rules (including to sec, cosec and cot); use $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$; and set up exponential models. Original teaching questions for specification items P3.4.1 to P3.4.4; the exam-style part adapts reviewed WMA13 June 2025 demand.

1 Key knowledge and worked examples

$$\frac{d}{dx}\sin kx = k\cos kx \qquad \frac{d}{dx}\cos kx = -k\sin kx \qquad \frac{d}{dx}\tan kx = k\sec^2 kx$$

$$\frac{d}{dx}e^{kx} = ke^{kx} \qquad \frac{d}{dx}\ln x = \frac{1}{x}$$

$$\text{product } (uv)' = u'v + uv' \qquad \text{quotient } \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \qquad \text{chain } \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

If x is given in terms of y, flip: $\frac{dy}{dx} = 1\Big/\frac{dx}{dy}$.

Worked example – product and chain together (adapted from Edexcel IAL WMA13 June 2025 Q7). Differentiate $y = e^{-x^2}\sin 3x$.

- $u = e^{-x^2}$: chain with inner $-x^2 \Rightarrow u' = -2x\,e^{-x^2}$.
- $v = \sin 3x \Rightarrow v' = 3\cos 3x$. Product rule:

$$\frac{dy}{dx} = -2xe^{-x^2}\sin 3x + 3e^{-x^2}\cos 3x = e^{-x^2}\big(3\cos 3x - 2x\sin 3x\big)$$

- Check at $x = 0$: $\frac{dy}{dx} = 3$; the curve leaves the origin like $\sin 3x$ (gradient 3) ✓.

Worked example – implicit flip. Find $\frac{dy}{dx}$ for $x = \sin 3y$.

- Differentiate the given form with respect to y:

$$\frac{dx}{dy} = 3\cos 3y \qquad \Rightarrow \qquad \frac{dy}{dx} = \frac{1}{3\cos 3y}$$

- Check at $y = 0$: $\frac{dy}{dx} = \frac{1}{3}$, and near the origin $x \approx 3y$, so $y \approx x/3$ (gradient $\frac{1}{3}$) ✓.

2 Practice

2.1 Differentiate $y = \sin 5x$. [1]

2.2 Differentiate $y = e^{-3x}$. [1]

2.3 Differentiate $y = x^2\cos x$ using the product rule. [3]

2.4 Differentiate $y = \frac{e^x}{x}$ using the quotient rule. [3]

2.5 Differentiate $y = \ln(4x^3 + 1)$ using the chain rule. [2]

2.6 Find $\frac{dy}{dx}$ for $x = e^{2y}$. [3]

3 Exam-style questions

3.1 *(Adapted from Edexcel IAL WMA13 June 2025 Q7.)* The curve C has equation $y = \mathrm{e}^{-x^2}\sin 3x$ for $0 \le x \le 3$. Show, using calculus, that the x-coordinates of the stationary points of C satisfy

$$x = \tfrac{1}{3}\arctan\frac{3}{2x}$$

[4]

3.2 A cylindrical tank drains so that the volume of water is $V = 500\mathrm{e}^{-0.1t}$ litres after t minutes. Find the rate at which water leaves at $t = 10$, and the time at which the rate is 5 litres per minute. [5]