

Solutions

2.1 $\frac{(2x-1)(x+3)}{(x-1)(x+3)} = \frac{2x-1}{x-1}$ ($x \neq 1, -3$). Check $x = 2$: original $\frac{(3)(5)}{(1)(5)} = 3$; simplified $\frac{3}{1} = 3$ ✓.

2.2 From $f(0) = 0$ increasing toward the limit 1: $0 \leq f(x) < 1$.

2.3 $g(3) = 7$ first, then $f(7) = 49$: $fg(3) = 49$. In fg , g acts first.

2.4 $y = 3x - 5 \Rightarrow x = \frac{y+5}{3}$, so $f^{-1}(x) = \frac{x+5}{3}$. Check: $f\left(\frac{x+5}{3}\right) = x + 5 - 5 = x$ ✓.

2.5 $y(x-2) = x+1 \Rightarrow x(y-1) = 2y+1 \Rightarrow x = \frac{2y+1}{y-1}$, so $g^{-1}(x) = \frac{2x+1}{x-1}$ ($x \neq 1$). Check: $g\left(\frac{2x+1}{x-1}\right) = \frac{(2x+1) + (x-1)}{(2x+1) - 2(x-1)} = \frac{3x}{3} = x$ ✓.

3.1 (a) $0 \leq f(x) < \frac{2}{3}$ (increasing from $f(0) = 0$ toward the horizontal limit $\frac{2}{3}$). (b) $y(3x+1) = 2x \Rightarrow x(3y-2) = -y$, so $f^{-1}(x) = \frac{x}{2-3x}$. (c) $\frac{x}{2-3x} = \frac{2x}{3x+1} \Rightarrow x(3x+1) = 2x(2-3x) \Rightarrow x(9x-3) = 0$, so $x = 0$ or $x = \frac{1}{3}$. Check $x = \frac{1}{3}$: $f\left(\frac{1}{3}\right) = \frac{1}{3}$ ✓.

3.2 $x^2 + x - 2 = (x+2)(x-1)$: $\frac{5x+1}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$ gives $5x+1 = A(x-1) + B(x+2)$. At $x = 1$: $6 = 3B$, $B = 2$; at $x = -2$: $-9 = -3A$, $A = 3$.

$$\frac{5x+1}{x^2+x-2} = \frac{3}{x+2} + \frac{2}{x-1}$$

Setting equal to 1: $5x+1 = x^2+x-2 \Rightarrow x^2-4x-3 = 0 \Rightarrow x = 2 \pm \sqrt{7}$. Check $x = 2 + \sqrt{7}$: denominator $\neq 0$ ✓.

Local review guidance: correct order for composition or correct solving-for- x structure; exact inverse/range; verification by composing or by a test value. This is authored classroom guidance.