

2.4 Sequences and series

Name: _____ Class: _____ Date: _____

Total: 23 marks

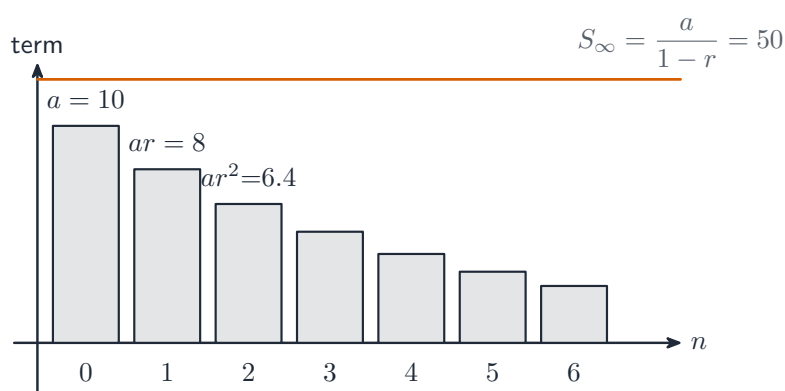
KEY WORDS arithmetic sequence • geometric sequence
 • sum to infinity • periodic • increasing

Objective

Work with sequences given by formulas or by $x_{n+1} = f(x_n)$, arithmetic and geometric series (including S_n , S_∞ , Σ notation and log-based term finding), and classify sequences as increasing, decreasing or periodic. Original teaching questions for specification items P2.4.1 to P2.4.4; the exam-style part adapts reviewed WMA12 June 2025 demand.

1 Key knowledge and worked examples

- **Arithmetic** : $a_n = a + (n-1)d$; $S_n = \frac{n}{2}\{2a + (n-1)d\}$; $\sum_{r=1}^n r = \frac{n(n+1)}{2}$. The sum formula's proof (add S_n written forwards and backwards) should be known.
- **Geometric** : $a_n = ar^{n-1}$; $S_n = \frac{a(1-r^n)}{1-r}$; $S_\infty = \frac{a}{1-r}$ when $|r| < 1$.
- Classify from the terms: **increasing** , **decreasing** , **periodic** (the sequence repeats with a fixed period).



Each bar is $r = 0.8$ times the one before. With $|r| < 1$ the bars shrink fast enough that the total settles at a finite limit.

$|r| < 1$ shrinks the terms; the total settles at S_∞ .

Worked example – arithmetic with negative difference. An arithmetic series starts 12, 9, 6, ... Find the 20th term and S_{20} .

- $a = 12$, $d = -3$:

$$a_{20} = 12 + 19(-3) = -45 \quad S_{20} = \frac{20}{2}\{24 + 19(-3)\} = 10(-33) = -330$$

- Check: the terms fall by 3 each step; the sum of a series heading negative is negative ✓.

Worked example – logs to find a term. A geometric series has $a = 48$ and $r = \frac{1}{4}$. Find the smallest n with $a_n < 0.001$.

$$48 \left(\frac{1}{4}\right)^{n-1} < 0.001 \implies \left(\frac{1}{4}\right)^{n-1} < \frac{1}{48\,000}$$

$$(n-1) \log \frac{1}{4} < -\log 48\,000 \implies n-1 > \frac{\log 48\,000}{\log 4} = 7.57\dots \implies n = 9$$

- Dividing by $\log \frac{1}{4}$ (negative) flips the inequality. Check: $n = 9$ gives $48/4^8 = 0.000732 < 0.001$ ✓; $n = 8$ gives $48/4^7 = 0.00293 > 0.001$ ✓ (correctly excluded).

2 Practice

2.1 For $u_n = 3n - 2$, write down the 10th term. [1]

2.2 For the arithmetic series $7 + 11 + 15 + \dots$, find S_{15} . [3]

2.3 A geometric sequence begins 8, 12, 18, ... Find its 10th term to 3 significant figures. [3]

2.4 A geometric series has $S_\infty = 60$ and $r = \frac{1}{2}$. Find a . [2]

2.5 The sequence defined by $x_{n+1} = 2x_n - 1$ with $x_1 = 3$: write down the first four terms and classify the sequence. [2]

2.6 The sequence with $x_{n+1} = -x_n$ and $x_1 = 4$: write down the first four terms and state the period. [2]

3 Exam-style questions

3.1 (*Adapted from Edexcel IAL WMA12 June 2025 Q7.*) A geometric series begins $18 + 12 + 8 + \dots$

(a) Find the sum to infinity of the series.

(b) Given that the k th term of the series is less than 0.01, use algebra to find the smallest possible value of k . [5]

3.2 (*Adapted from the same question.*) An arithmetic series has first term 500 and common difference -7 . Find the greatest possible value of S_n . [5]
