Solutions

2.1 $(x+4)^2+(y-1)^2=9$: centre $(-4,1)$, radius 3.

2.2 $(x-2)^2+(y+5)^2=13$.

2.3 $(5-1)^2+(2-3)^2=16+1=17$ – yes, the point lies on the circle.

2.4 Centre: midpoint $=(1,4)$. Diameter squared: $8^2+6^2=100$, so $r^2=25$.

$$(x-1)^2+(y-4)^2=25$$

Check both endpoints: $16+9=25$ ✓ twice.

2.5 The radius to $(3,1)$ has gradient $\frac{1}{3}$; the tangent's gradient is -3:

$$y-1=-3(x-3) \implies y=-3x+10$$

Check: the tangent passes $(3,1)$ ✓, and $-3\times\frac{1}{3}=-1$ (perpendicular) ✓.

3.1 (a) Centre: $\left(\frac{-1+7}{2},\frac{-3+1}{2}\right)=(3,-1)$. Diameter squared: $8^2+4^2=80$, so $r^2=20$:

$$(x-3)^2+(y+1)^2=20$$

Check an endpoint: $(-4)^2+2^2=20$ ✓. (b) Nearest the x-axis means straight below the centre (the centre is 1 unit below the axis, less than $r=\sqrt{20}\approx 4.47$, so the circle crosses the axis; the lowest point is r below the centre):

$$(3,\ -1-\sqrt{20})=(3,\ -1-2\sqrt{5})$$

3.2 The centre is $(4,2)$. The perpendicular from $(4,2)$ to the vertical chord $x=0$ is the horizontal line $y=2$, meeting AB at $(0,2)$. The midpoint of $A(0,5)$ and $B(0,-1)$ is $\left(0,\frac{5+(-1)}{2}\right)=(0,2)$ – the same point, so the perpendicular bisects the chord ✓.

Local review guidance: midpoint/completing-the-square method; exact centre, radius and equation; property applied with the right perpendicular. This is authored classroom guidance.