2.10 The trapezium rule

Name: _____ Class: _____ Date: _____ **Total: 22 marks**

KEY WORDS trapezium rule • underestimate • overestimate • strip

Objective

Approximate areas with the trapezium rule from a table of values, understand how strip width and curvature control the error, and scale known integral estimates. Original teaching questions for specification item P2.8.3; the exam-style part adapts reviewed WMA12 June 2025 demand.

1 Key knowledge and worked examples

With n strips of width h between x_0 and x_n, and heights $y_0, y_1, \ldots, y_n$:

$$\int_a^b y\,dx \approx \frac{h}{2}\big[y_0 + y_n + 2(y_1 + y_2 + \cdots + y_{n-1})\big]$$

- More strips (smaller h) improve the estimate.
- A curve bending upwards (**convex**, like x^2) leaves the straight strips **above** it: **overestimate** . A curve bending downwards (**concave**, like $\sqrt{x}$) leaves them **below**: **underestimate** .

Worked example. Estimate $\displaystyle\int_0^2 \sqrt{4-x^2}\,dx$ with 4 strips.

- $h = 0.5$; heights at $x = 0, 0.5, 1, 1.5, 2$: 2, 1.9365, 1.7321, 1.3229, 0.

$$\int_0^2 \sqrt{4-x^2}\,dx \approx \frac{0.5}{2}\big[2 + 0 + 2(1.9365 + 1.7321 + 1.3229)\big] = 0.25 \times 11.983 = 3.00$$

- The curve is a quarter circle of radius 2, whose true area is $\frac{1}{4}\pi(2)^2 = \pi \approx 3.14$: an **underestimate**, because the quarter circle is concave (bends downwards), so every chord lies below the arc.

2 Practice

2.1 Estimate $\displaystyle\int_0^4 x^2\,dx$ with 4 strips. [3]

2.2 Estimate $\displaystyle\int_1^3 \frac{1}{x}\,dx$ with 2 strips, and state whether your answer is an over- or under-estimate. [3]

2.3 For a curve that is convex (bends upwards), state whether the trapezium rule over- or underestimates the true area, and why. [2]

2.4 Estimate $\displaystyle\int_0^1 \sqrt{x}\,dx$ with 4 strips. [3]

2.5 State two ways to improve a trapezium-rule estimate. [2]

3 Exam-style questions

3.1 *(Adapted from Edexcel IAL WMA12 June 2025 Q5.)* The trapezium rule with five y-values and strip width $h = 1.5$ gives the estimate

$$\int_1^5 f(x)\,dx \approx 12.4$$

Using this result and making your method clear, estimate

(a) $\displaystyle\int_1^5 \frac{f(x)}{2}\,dx$

(b) $\displaystyle\int_1^5 \left(\frac{f(x)}{4} + 3x\right) dx$ [4]

3.2 *(Structure adapted from the same paper.)* Use the trapezium rule with 8 strips to estimate $\displaystyle\int_0^4 \sqrt{x}\,dx$, and compare your answer with the exact value $\frac{16}{3}$. [5]