Solutions

2.1 $h = 1$; heights $0, 1, 4, 9, 16$: $\frac{1}{2}\big[0 + 16 + 2(1 + 4 + 9)\big] = \frac{1}{2}(44) = 22$. (True value $64/3 \approx 21.3$: an overestimate, as x^2 is convex and the chords sit above.)

2.2 $h = 1$; heights at $x = 1, 2, 3$: $1, 0.5, 0.3333$: $\frac{1}{2}\big[1 + 0.3333 + 2(0.5)\big] = \frac{4}{3} = 1.333$. $y = 1/x$ is convex (bends upwards), so the chords lie above the curve: an **overestimate** (true value $\ln 3 = 1.099$).

2.3 Overestimate: a convex curve bends upwards, so every chord (strip top) sits above the curve and the strips count extra area between chord and curve.

2.4 $h = 0.25$; heights $0, 0.5, 0.7071, 0.8660, 1$: $0.125\big[0 + 1 + 2(2.0731)\big] = 0.125 \times 5.146 = 0.643$.

2.5 Use more strips (smaller h); the rule is also exact when the graph is straight, so flatter regions estimate better – in general, more strips and checking the curvature direction.

3.1 (a) Constants factor out of integrals:

$$\int_1^5 \frac{f(x)}{2}\,dx = \frac{1}{2} \times 12.4 = 6.2$$

(b) Split the integral and scale each part:

$$\int_1^5 \left(\frac{f(x)}{4} + 3x\right) dx = \frac{1}{4}(12.4) + 3\left[\frac{x^2}{2}\right]_1^5 = 3.1 + 3(12 - 0.5) = 3.1 + 34.5 = 37.6$$

Check: $\frac{5^2 - 1^2}{2} = 12$ ✓.

3.2 $h = 0.5$; heights at $x = 0, 0.5, \ldots, 4$: $0, 0.7071, 1, 1.2247, 1.4142, 1.5811, 1.7321, 1.8708, 2$.

$$\frac{0.5}{2}\big[0{+}2{+}2(0.7071{+}1{+}1.2247{+}1.4142{+}1.5811{+}1.7321{+}1.8708)\big] = 0.25\big[2{+}19.060\big] = 5.265$$

So the estimate is 5.27 (3 s.f.), below the exact $\frac{16}{3} = 5.333$: an underestimate, because $\sqrt{x}$ is concave (it rises steeply then flattens), so each chord lies below the curve and the strips undercount.

Local review guidance: correct trapezium bracket structure with $h/2$; arithmetic to the requested accuracy; error direction tied to the curvature. This is authored classroom guidance.