8.1 Binomial and Poisson distributions

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

KEY WORDS binomial distribution • Poisson distribution

Binomial and Poisson distributions

Objective

State the modelling assumptions, solve mean/variance pairs for the parameters, scale Poisson rates with time, and use the Poisson approximation to the binomial (objectives 1.1–1.3).

1 Key knowledge and worked examples

Models. Binomial: n independent trials, constant probability p; $E = np$, Var $= np(1-p)$. Poisson: events singly, independently, at a constant rate; mean and vararianceance both λ; the rate scales with time. Poisson approximates binomial when n is large and p small ($\lambda = np$).

Worked example – parameters then approximations (WST02 June 2025 Q3). Blue sweets per small bag: mean 3, variance 2.55, so $np = 3$ and $np(1-p) = 2.55$ give $p = 0.15$, $n = 20$. Medium bag: $B(40, 0.2)$, $P(X > 8) = 1 - P(X \leq 8) = 0.407$. Large bag: $n = 80$ large, $p = 0.1$ small, so $X \approx \text{Po}(8)$ and $P(X \geq 12) = 0.112$.

2 Practice

2.1 $X \sim B(60, 0.25)$. Write down $E(X)$ and $\text{Var}(X)$. [2]

2.2 Emails arrive at an average rate of 6 per 20 minutes. State the distribution of the number of emails in one hour and write down its variance. [2]

2.3 A binomial variable has mean 4 and variance 2.4. Find n and p. [3]

3 Exam-style questions

3.1 *(Edexcel IAL WST02 June 2025 Q3.)* The proportion of blue sweets is p. Each small bag contains a random sample of n sweets; the mean number of blue sweets per small bag is 3 and the variance is 2.55. (a) Find n and p. (b) The proportion of green sweets is 0.2 and a medium bag contains 40 sweets; find the probability that a medium bag contains more than the expected number of green sweets. (c) The proportion of yellow sweets is 0.1 and a large bag contains 80 sweets; (i) using a Poisson approximation, find the probability that a large bag contains at least 12 yellow sweets, (ii) justify the use of the approximation. [10]

3.2 *(Original.)* Faults occur in a roll of fabric at a mean rate of 1.5 per 10 metres. A 40-metre roll is inspected. (a) Find the probability of exactly 5 faults. (b) Find the probability of at most 2 faults. (c) State two conditions for the Poisson model to be appropriate. [6]

Solutions

2.1 $E(X) = 15$; $\mathrm{Var}(X) = 60(0.25)(0.75) = 11.25$.

2.2 Po(18) per hour; variance 18.

2.3 $np = 4$, $np(1-p) = 2.4$: $4(1-p) = 2.4$ gives $p = 0.4$; $n = 4/0.4 = 10$.

3.1 (a) $np = 3$, $np(1-p) = 2.55$: dividing, $1-p = 0.85$, so $p = 0.15$ and $n = \frac{3}{0.15} = 20$. (b) $X \sim B(40, 0.2)$ with $E(X) = 8$: $P(X > 8) = 1 - P(X \leq 8) = 1 - 0.5931 = 0.407$. (c)(i) $Y \approx \mathrm{Po}(80 \times 0.1) = \mathrm{Po}(8)$: $P(Y \geq 12) = 1 - P(Y \leq 11) = 1 - 0.888 = 0.112$. (ii) $n = 80$ is large and $p = 0.1$ is small.

3.2 (a) Po(6): $P(X = 5) = e^{-6}\frac{6^5}{5!} = 0.1606$ (0.161). (b) $P(X \leq 2) = e^{-6}(1 + 6 + 18) = 0.0620$ (0.062). (c) Faults occur singly, independently, at a constant average rate.

Local review guidance: mean/variance pair divided to eliminate n first; Poisson rates scaled by the time ratio before any table lookup; assumptions stated in context. This is authored classroom guidance.