

6.5 Discrete random variables

Name: _____ Class: _____ Date: _____

Total: 23 marks

KEY WORDS discrete random variable • expectation

Discrete random variables

Objective

Use probability functions and cumulative functions of discrete random variables, compute $E(X)$, $E(X^2)$, $\text{Var}(X)$, apply linear coding, and quote the discrete uniform results (objectives 5.1–5.4).

1 Key knowledge and worked examples

Routine. Probabilities sum to 1 (find missing values). $F(x_0) = P(X \leq x_0)$. Solve inequalities for X BEFORE summing probabilities. $E(X) = \sum xp(x)$, $E(X^2) = \sum x^2p(x)$, $\text{Var}(X) = E(X^2) - [E(X)]^2$; $E(aX + b) = aE(X) + b$, $\text{Var}(aX + b) = a^2\text{Var}(X)$. Uniform on $1..n$: $E(X) = \frac{n+1}{2}$, $\text{Var}(X) = \frac{n^2-1}{12}$.

Worked example – the full routine (WST01 June 2025 Q1). X on 1–4 with probabilities 0.3, 0.2, 0.35, 0.15: $F(3.5) = 0.85$; $3X - 3 < X + 2 \Rightarrow X < 2.5 \Rightarrow P = 0.5$; $E(X) = 2.35$; $E(X^2) = 6.65$; $\text{Var}(X) = 1.1275$; $\text{Var}(5 - 2X) = 4(1.1275) = 4.51$.

2 Practice

2.1 X has $P(X = x) = \frac{kx}{6}$ for $x = 1, 2, 3$. Find k , $E(X)$ and $\text{Var}(X)$. [4]

2.2 A fair die is rolled once. Using the uniform results, write down $E(X)$ and $\text{Var}(X)$. [2]

2.3 Y has $E(Y) = 4$ and $\text{Var}(Y) = 2.5$. Find $E(3Y - 2)$ and $\text{Var}(3Y - 2)$. [2]

3 Exam-style questions

3.1 (*Edexcel IAL WST01 June 2025 Q1.*) The discrete random variable X has probability distribution

x	1	2	3	4
$P(X = x)$	0.3	0.2	0.35	0.15

(a) Find $F(3.5)$. (b) Find $P(3X - 3 < X + 2)$. (c) Find $E(X)$. (d) Show that $\text{Var}(X) = 1.1275$. (e) Find $\text{Var}(5 - 2X)$. [9]

3.2 (*Original.*) A game scores X where $P(X = 0) = 0.25$, $P(X = 1) = 0.5$ and $P(X = 4) = 0.25$. The winnings are $W = 10X - 3$. (a) Find $E(W)$ and $\text{Var}(W)$. (b) Find $P(W > E(W))$. [6]

Solutions

2.1 $\frac{k}{6}(1+2+3) = 1$ gives $k = 1$: so the probabilities are $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}$. $E(X) = \frac{1+4+9}{6} = \frac{14}{6}$, $E(X^2) = \frac{1+8+27}{6} = \frac{36}{6} = 6$; $\text{Var}(X) = 6 - \left(\frac{14}{6}\right)^2 = 6 - \frac{196}{36} = \frac{5}{9}$.

2.2 Uniform on 1–6: $E(X) = \frac{7}{2} = 3.5$; $\text{Var}(X) = \frac{36-1}{12} = \frac{35}{12} \approx 2.92$.

2.3 $E(3Y - 2) = 3(4) - 2 = 10$; $\text{Var}(3Y - 2) = 3^2(2.5) = 22.5$.

3.1 (a) $F(3.5) = P(X \leq 3.5) = 0.3 + 0.2 + 0.35 = 0.85$. (b) $3X - 3 < X + 2$ simplifies to $2X < 5$, i.e. $X < 2.5$, so $P = P(1) + P(2) = 0.5$. (c) $E(X) = 0.3 + 0.4 + 1.05 + 0.6 = 2.35$. (d) $E(X^2) = 0.3 + 0.8 + 3.15 + 2.4 = 6.65$; $\text{Var}(X) = 6.65 - 2.35^2 = 6.65 - 5.5225 = 1.1275$ ✓. (e) $\text{Var}(5 - 2X) = (-2)^2(1.1275) = 4.51$.

3.2 (a) $E(X) = 0 + 0.5 + 1 = 1.5$; $E(W) = 10(1.5) - 3 = 12$. $E(X^2) = 0 + 0.5 + 4 = 4.5$; $\text{Var}(X) = 4.5 - 2.25 = 2.25$; $\text{Var}(W) = 100(2.25) = 225$. (b) $W > 12 \Leftrightarrow 10X - 3 > 12 \Leftrightarrow X > 1.5 \Leftrightarrow X = 4$: $P = 0.25$.

Local review guidance: probabilities totalled to 1 before anything else; inequality solved for X first; variance via $E(X^2) - E(X)^2$ and coding scalars squared. This is authored classroom guidance.