

4.6 Vectors

Name: _____ Class: _____ Date: _____

Total: 21 marks

KEY WORDS magnitude • unit vector • position vector
• scalar product

Objective

Work with vectors in two and three dimensions: magnitude and unit vectors, addition and scalar multiplication with their geometric meanings, position vectors, and angles from the scalar product. Original teaching questions for specification items P4.7.1 to P4.7.7; the exam-style part adapts reviewed WMA14 June 2025 demand.

1 Key knowledge and worked examples

- Column and **ijk** forms: $a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$.
- **Magnitude** : $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$; **unit vector** $\hat{\mathbf{a}} = \mathbf{a}/|\mathbf{a}|$.
- **Position vector** : $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$.
- **Scalar product** : $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{a}||\mathbf{b}|\cos\theta$. Parallel vectors are scalar multiples; perpendicular ones have zero scalar product.

Worked example – magnitudes and a unit vector. $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$.

- $|\mathbf{a}| = \sqrt{4 + 1 + 4} = 3$, so the unit vector is $\frac{1}{3}(2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$.
- Check: its magnitude is $\frac{1}{3}\sqrt{9} = 1$ ✓.

Worked example – a 45° angle (Edexcel IAL WMA14 June 2025 Q6). $A = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$, $B = 5\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $C = 3\mathbf{i} + \alpha\mathbf{j} + 5\mathbf{k}$, angle $BAC = 45^\circ$; find α .

- $\overrightarrow{AB} = 4\mathbf{i} + 4\mathbf{k}$ with $|\overrightarrow{AB}| = \sqrt{32}$; $\overrightarrow{AC} = 2\mathbf{i} + (\alpha - 2)\mathbf{j} + 8\mathbf{k}$; scalar product = $8 + 0 + 32 = 40$.

$$\cos 45^\circ = \frac{40}{\sqrt{32}\sqrt{68 + (\alpha - 2)^2}} \implies \sqrt{68 + (\alpha - 2)^2} = \frac{40\sqrt{2}}{4\sqrt{2}} = 10 \implies (\alpha - 2)^2 = 32$$

$$\alpha = 2 \pm 4\sqrt{2}$$

- Check: $|AB||AC|\cos 45^\circ = 4\sqrt{2} \times 10 \times \frac{1}{\sqrt{2}} = 40$ ✓.

2 Practice

2.1 Find $|3\mathbf{i} + 4\mathbf{j}|$ and the unit vector in that direction. [2]

2.2 For $P(1, 2, 3)$ and $Q(4, 6, 3)$, find $\overrightarrow{PQ}$ and its magnitude. [3]

2.3 Determine whether $\mathbf{a} = 2\mathbf{i} + (-1)\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = (-4)\mathbf{i} + 2\mathbf{j} + (-6)\mathbf{k}$ are parallel. [2]

2.4 Find the angle between $1\mathbf{i} + 1\mathbf{k}$ and $1\mathbf{j} + 1\mathbf{k}$, giving an exact value. [4]

3 Exam-style questions

3.1 (*Adapted from Edexcel IAL WMA14 June 2025 Q6.*) Relative to O : A has position vector $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and B has position vector $3\mathbf{i} + \mathbf{j} + 6\mathbf{k}$. The point C has position vector $2\mathbf{i} + \beta\mathbf{j} + 4\mathbf{k}$, and angle BAC is 60° . Find the exact possible values of β . [6]

3.2 The line segment RS has midpoint $M(1, 1, 0)$; R has position vector $3\mathbf{i} - \mathbf{j} + 4\mathbf{k}$. Find $\overrightarrow{RS}$ and the distance RS . [4]

Solutions

2.1 Magnitude 5; unit vector $\frac{1}{5}(3\mathbf{i} + 4\mathbf{j})$.

2.2 $\overrightarrow{PQ} = 3\mathbf{i} + 4\mathbf{j}$, magnitude 5.

2.3 $\mathbf{b} = -2\mathbf{a}$: yes, parallel (one is a scalar multiple of the other).

2.4 Dot product $= 0 + 0 + 1 = 1$; magnitudes $\sqrt{2}$ each:

$$\cos \theta = \frac{1}{\sqrt{2} \cdot \sqrt{2}} = \frac{1}{2} \implies \theta = 60^\circ$$

3.1 $\overrightarrow{AB} = 2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$, $|AB| = \sqrt{24}$; $\overrightarrow{AC} = 1\mathbf{i} + (\beta + 1)\mathbf{j} + 2\mathbf{k}$; dot product $= 2 + 2(\beta + 1) + 8 = 2\beta + 12$.

$$\cos 60^\circ = \frac{2\beta + 12}{\sqrt{24}\sqrt{1 + (\beta + 1)^2 + 4}} = \frac{1}{2}$$

$$2(2\beta + 12) = \sqrt{24}\sqrt{(\beta + 1)^2 + 5} \implies 16(\beta + 6)^2 = 24((\beta + 1)^2 + 5)$$

$$2(\beta + 6)^2 = 3((\beta + 1)^2 + 5) \implies 2\beta^2 + 24\beta + 72 = 3\beta^2 + 6\beta + 18$$

$$\beta^2 - 18\beta - 54 = 0 \implies \beta = \frac{18 \pm \sqrt{540}}{2} = 9 \pm 3\sqrt{15}$$

- Check $\beta = 9 + 3\sqrt{15} \approx 20.62$: dot $= 53.2$; $|AC| = \sqrt{1 + 467.4 + 4} = 21.7$; $\cos \theta = 53.2/(4.90 \times 21.7) = 0.500 = \frac{1}{2}$ ✓ exactly.

3.2 $\overrightarrow{OR} = 3\mathbf{i} + (-1)\mathbf{j} + 4\mathbf{k}$; $\overrightarrow{OM} = \frac{1}{2}(\overrightarrow{OR} + \overrightarrow{OS}) \Rightarrow \overrightarrow{OS} = 2\overrightarrow{OM} - \overrightarrow{OR} = (-1)\mathbf{i} + 3\mathbf{j} + (-4)\mathbf{k}$.

$$\overrightarrow{RS} = \overrightarrow{OS} - \overrightarrow{OR} = (-4)\mathbf{i} + 4\mathbf{j} + (-8)\mathbf{k}, \quad RS = \sqrt{16 + 16 + 64} = \sqrt{96} = 4\sqrt{6}$$

- Check: M is the midpoint of R and S : $\frac{1}{2}(3 + (-1)) = 1$, $\frac{1}{2}(-1 + 3) = 1$, $\frac{1}{2}(4 + (-4)) = 0$ ✓.

Local review guidance: correct $\overrightarrow{AB}$, $\overrightarrow{AC}$ and scalar product; exact β or distance; angle equation set up with magnitudes. This is authored classroom guidance.