

## 3.2 Modulus and combined transformations

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 21 marks

**KEY WORDS** modulus • combined transformations

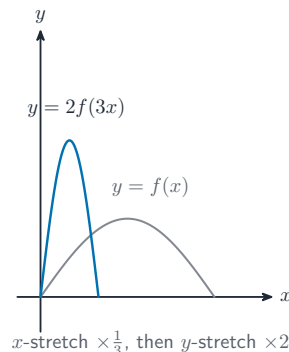
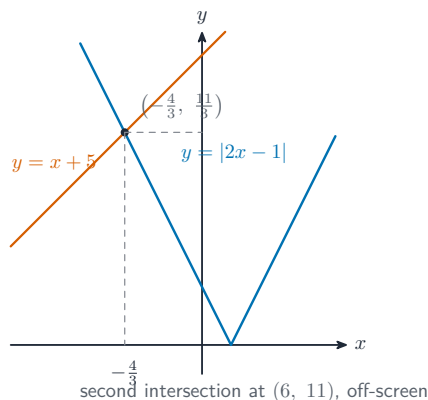
### Objective

Sketch and solve equations with the modulus function, sketch  $y = |f(x)|$  and  $y = f(|x|)$  from a given graph, and combine two transformations on one graph. Original teaching questions for specification items P3.1.3 and P3.1.4.

### 1 Key knowledge and worked examples

The **modulus**  $|x|$  equals  $x$  when  $x \geq 0$  and  $-x$  when  $x < 0$ , so  $|ax + b|$  never goes below the  $x$ -axis: its graph is the line  $y = ax + b$  with the below-axis part reflected up. Solve modulus equations **by cases**, checking every candidate against its case's condition.

From  $y = f(x)$  you should also sketch  $y = |f(x)|$  (reflect below-axis parts up) and  $y = f(|x|)$  (keep the  $x \geq 0$  half and mirror it to the left). **Combined transformations** stack the P1 rules: e.g.  $y = 2f(3x)$  first stretches parallel to the  $x$ -axis by  $\frac{1}{3}$ , then parallel to the  $y$ -axis by 2.



*The V vertex sits at the root of the inside expression.*

**Worked example – modulus equation** (the specification's own example). Solve  $|2x - 1| = x + 5$ .

- Case  $x \geq \frac{1}{2}$ :  $2x - 1 = x + 5 \implies x = 6$  (valid:  $6 \geq \frac{1}{2}$  ✓).
- Case  $x < \frac{1}{2}$ :  $1 - 2x = x + 5 \implies x = -\frac{4}{3}$  (valid:  $-\frac{4}{3} < \frac{1}{2}$  ✓).
- Solutions:  $x = 6$  and  $x = -\frac{4}{3}$ . Check:  $|11| = 11$  ✓ and  $|- \frac{11}{3}| = \frac{11}{3} = -\frac{4}{3} + 5$  ✓.

### 2 Practice

2.1 Sketch  $y = |x - 2|$ , marking the vertex. [2]

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2.2 Solve  $|x - 3| = 5$ . [2]

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2.3 Solve  $|2x + 1| = 7$ . [2]

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2.4 Describe the transformations mapping  $y = f(x)$  onto  $y = 3f(x) - 2$ . [2]

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2.5 Given  $y = f(x)$  passes through  $(2, 5)$ , state the image of this point on  $y = f(2x) + 3$ . [3]

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3 Exam-style questions

3.1 Solve  $|x^2 - 4| = 3x + 2$ , giving exact answers and checking each one. [5]

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3.2 The graph of  $y = f(x)$  has a maximum at  $(1, 4)$  and crosses the  $x$ -axis at  $x = -1$  and  $x = 3$ . Sketch, on separate axes,  $y = |f(x)|$  and  $y = f(|x|)$ , labelling key points. [5]

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## Solutions

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**2.1** A V with vertex at  $(2, 0)$ , arms of gradient  $\pm 1$ .

**2.2**  $x - 3 = \pm 5$ :  $x = 8$  or  $x = -2$ .

**2.3**  $2x + 1 = \pm 7$ :  $x = 3$  or  $x = -4$ .

**2.4** Stretch parallel to the  $y$ -axis, factor 3, then translation  $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$  (down 2).

**2.5**  $y = f(2x)$  halves  $x$ -coordinates:  $(2, 5) \rightarrow (1, 5)$ ; adding 3 moves it up:  $(1, 8)$ .

**3.1** Case  $x^2 - 4 \geq 0$  ( $|x| \geq 2$ ):  $x^2 - 4 = 3x + 2 \implies x^2 - 3x - 6 = 0 \implies x = \frac{3 \pm \sqrt{33}}{2}$ . The  $+$  root  $= 4.37 \geq 2$  ✓; the  $-$  root  $= -1.37$  fails  $|x| \geq 2$  ✗. Case  $x^2 - 4 < 0$  ( $|x| < 2$ ):  $4 - x^2 = 3x + 2 \implies x^2 + 3x - 2 = 0 \implies x = \frac{-3 \pm \sqrt{17}}{2}$ . The  $+$  root  $= 0.56$ , inside  $(-2, 2)$  ✓; the  $-$  root  $= -3.56$  fails ✗. Solutions:  $x = \frac{3 + \sqrt{33}}{2}$  and  $x = \frac{-3 + \sqrt{17}}{2}$ . Check the second:  $|0.31 - 4| = 3.69$  and  $3(0.56) + 2 = 3.69$  ✓.

**3.2**  $y = |f(x)|$ : for  $-1 \leq x \leq 3$  the original sits below zero between the roots, so reflect that dip above the axis – maxima at  $(1, 4)$  and the reflected dip peaking at  $(1, 4)$  only if the minimum was exactly  $-4$ ; otherwise label the reflected minimum as  $|\min|$ , keeping the roots  $x = -1, 3$  on the axis.  $y = f(|x|)$ : keep the  $x \geq 0$  half (crossing at  $x = 3$ , falling from the maximum at  $x = 1$ ) and mirror it into  $x < 0$ , so the maximum appears at both  $x = \pm 1$  and the crossing at  $x = \pm 3$ .

Local review guidance: correct case split with its condition; exact valid roots; each root checked against its case. This is authored classroom guidance.