2.4 Sequences and series

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

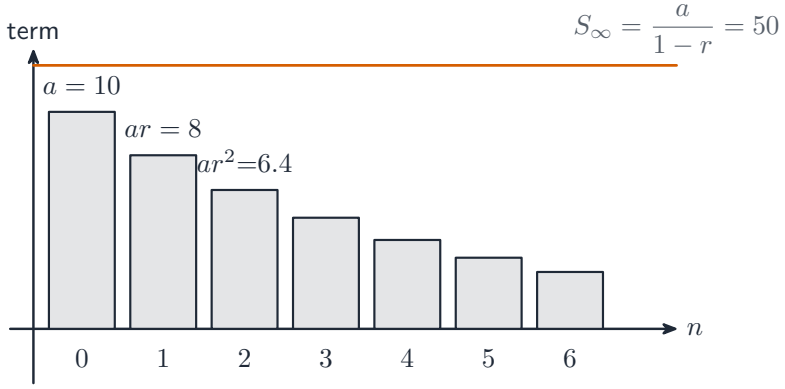
KEY WORDS arithmetic sequence • geometric sequence • sum to infinity • periodic • increasing

Objective

Work with sequences given by formulas or by $x_{n+1} = f(x_n)$, arithmetic and geometric series (including S_n, S_∞, Σ notation and log-based term finding), and classify sequences as increasing, decreasing or periodic. Original teaching questions for specification items P2.4.1 to P2.4.4; the exam-style part adapts reviewed WMA12 June 2025 demand.

1 Key knowledge and worked examples

- **Arithmetic** : $a_n = a+(n-1)d$; $S_n = \frac{n}{2}\{2a+(n-1)d\}$; $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$. The sum formula's proof (add S_n written forwards and backwards) should be known.
- **Geometric** : $a_n = ar^{n-1}$; $S_n = \dfrac{a(1-r^n)}{1-r}$; $S_\infty = \dfrac{a}{1-r}$ when $|r| < 1$.
- Classify from the terms: **increasing** , **decreasing** , **periodic** (the sequence repeats with a fixed period).



Each bar is $r = 0.8$ times the one before. With $|r| < 1$ the bars shrink fast enough that the total settles at a finite limit.

|r| < 1 shrinks the terms; the total settles at S∞.

Worked example – arithmetic with negative difference. An arithmetic series starts $12, 9, 6, \ldots$ Find the 20th term and S_{20}.

- $a = 12$, $d = -3$:

$$a_{20} = 12 + 19(-3) = -45 \qquad S_{20} = \tfrac{20}{2}\{24 + 19(-3)\} = 10(-33) = -330$$

- Check: the terms fall by 3 each step; the sum of a series heading negative is negative ✓.

Worked example – logs to find a term. A geometric series has $a = 48$ and $r = \frac{1}{4}$. Find the smallest n with $a_n < 0.001$.

$$48\left(\tfrac{1}{4}\right)^{n-1} < 0.001 \implies \left(\tfrac{1}{4}\right)^{n-1} < \frac{1}{48\,000}$$

$$(n-1)\log\tfrac{1}{4} < -\log 48\,000 \implies n - 1 > \frac{\log 48\,000}{\log 4} = 7.57\ldots \implies n = 9$$

- Dividing by $\log\frac{1}{4}$ (negative) flips the inequality. Check: $n = 9$ gives $48/4^8 = 0.000732 < 0.001$ ✓; $n = 8$ gives $48/4^7 = 0.00293 > 0.001$ ✓ (correctly excluded).

2 Practice

2.1 For $u_n = 3n - 2$, write down the 10th term. [1]

2.2 For the arithmetic series $7 + 11 + 15 + \cdots$, find S_{15}. [3]

2.3 A geometric sequence begins $8, 12, 18, \ldots$ Find its 10th term to 3 significant figures. [3]

2.4 A geometric series has $S_\infty = 60$ and $r = \frac{1}{2}$. Find a. [2]

2.5 The sequence defined by $x_{n+1} = 2x_n - 1$ with $x_1 = 3$: write down the first four terms and classify the sequence. [2]

2.6 The sequence with $x_{n+1} = -x_n$ and $x_1 = 4$: write down the first four terms and state the period. [2]

3 Exam-style questions

3.1 *(Adapted from Edexcel IAL WMA12 June 2025 Q7.)* A geometric series begins $18 + 12 + 8 + \cdots$

(a) Find the sum to infinity of the series.

(b) Given that the kth term of the series is less than 0.01, use algebra to find the smallest possible value of k. [5]

3.2 *(Adapted from the same question.)* An arithmetic series has first term 500 and common difference -7. Find the greatest possible value of S_n. [5]

Solutions

2.1 $u_{10} = 3(10) - 2 = 28$.

2.2 $a = 7$, $d = 4$: $S_{15} = \frac{15}{2}\{14 + 14 \times 4\} = 7.5 \times 70 = 525$.

2.3 $r = \frac{12}{8} = 1.5$: $a_{10} = 8(1.5)^9 = 8 \times 38.44 = 308$ (3 s.f.).

2.4 $a = S_\infty(1 - r) = 60 \times \frac{1}{2} = 30$.

2.5 $3, 5, 9, 17$: increasing (each term exceeds the last).

2.6 $4, -4, 4, -4$: periodic with period 2.

3.1 (a) $r = \frac{2}{3}$, $|r| < 1$: $S_\infty = \dfrac{18}{1 - 2/3} = 54$. (b) $18\left(\frac{2}{3}\right)^{k-1} < 0.01 \implies \left(\frac{2}{3}\right)^{k-1} < \frac{1}{1800}$. Taking logs and dividing by the negative $\log\frac{2}{3}$ flips the sign:

$$k - 1 > \frac{\log(1/1800)}{\log(2/3)} = 18.48\ldots \implies k = 20$$

Check: $18\left(\frac{2}{3}\right)^{19} = 0.0081 < 0.01$ ✓ and $18\left(\frac{2}{3}\right)^{18} = 0.0122 > 0.01$ ✓ (so 19 is excluded).

3.2 Terms stay positive while $500 - 7(n - 1) > 0$, i.e. $n \leq 72$; check the boundary: $u_{72} = 500 - 7(71) = 3 > 0$ and $u_{73} = -4 < 0$, so the maximum partial sum is

$$S_{72} = \tfrac{72}{2}\{1000 + 71(-7)\} = 36 \times 503 = 18\,108$$

Adding the negative 73rd term would reduce the total ✓.

Local review guidance: correct formula with a and r or d; exact or correctly rounded value; inequality direction preserved through negative-log division, or the boundary term checked. This is authored classroom guidance.