

Exercise walk-through

9.5

Linear programming

Pearson Edexcel · International A-Level · Mathematics

7. Critical path analysis

Critical paths

Precedence table to activity-on-arc network; dummies carry dependencies. Forward pass takes maxima (earliest event times), backward pass minima (latest). Float = latest finish – earliest start – duration; critical activities float zero. The example A(3), B(4), C(4), D(1), E(5) has critical path A–C–E = 12 days and D floats 2; Gantt bars run from earliest starts.

Check yourself

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Bubble and quick sorts pass by pass; first-fit, decreasing and full-bin placements; Kruskal and Prim (matrix form too); Dijkstra with permanence order and route recovery; odd-node pairings for the postman; TSP bounds both sides with nearest neighbour; dummies, forward and backward passes, floats, Gantt charts; LP formulation and vertex solutions with integer checks.

Exercises 2.1·2.3

2.1 A baker makes x plain and y iced buns. Flour: at most 200 kg, with plain buns using 2 kg per dozen and iced 4 kg per dozen. Time: at most 60 hours, with 1 hour per dozen plain and 1.5 hours per dozen iced. Write the linear program for maximum profit at 3 pounds per dozen plain and 5 pounds per dozen iced. [4]

2.2 For the constraints $x + y \leq 8$, $y \geq 2$, $x \leq 5$, $x, y \geq 0$, list the vertices of the feasible region. [3]

2.3 Minimise $C = 4x + 3y$ over the region of 2.2, using the vertex method. [3]

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2.1 Maximise $P = 3x + 5y$ subject to $2x + 4y \leq 200$ (flour), $x + 1.5y \leq 60$ (time), $x, y \geq 0$.

2.2 For the constraints $x + y \leq 8$, $y \geq 2$, $x \leq 5$, $x, y \geq 0$, list the vertices of the feasible region. [3]

2.2 The region is bounded by $y = 2$ below, $x = 5$ right and $x + y = 8$ above: vertices $(0, 2)$ (y -axis meets $y = 2$), $(0, 8)$ (y -axis meets $x + y = 8$), $(5, 3)$ ($x = 5$ meets $x + y = 8$) and $(5, 2)$ ($x = 5$ meets $y = 2$).

2.3 Minimise $C = 4x + 3y$ over the region of 2.2, using the vertex method. [3]

2.3 C at the vertices: $(0, 2)$: 6; $(0, 8)$: 24; $(5, 3)$: 29; $(5, 2)$: 26. Minimum $C = 6$ at $(0, 2)$.

Exercise 3.1

3.1 (*Original.*) A factory makes chairs (x) and tables (y). Each chair needs 2 hours of carpentry and each table 4 hours; at most 48 hours are available. Each chair needs 1 hour of finishing and each table 2 hours; at most 20 hours are available. Demand limits tables to at most 6. The profit is 40 pounds per chair and 70 pounds per table. (a) Formulate the linear program. (b) Solve graphically by the vertex method, listing every vertex and the profit there. (c) If chairs and tables must be whole units, confirm the optimal integer solution.

[10]

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3.1 (a) Maximise $P = 40x + 70y$ subject to $2x + 4y \leq 48$, $x + 2y \leq 20$, $y \leq 6$, $x, y \geq 0$.
 (b) The carpentry constraint $2x + 4y \leq 48$ is equivalent to $x + 2y \leq 24$, which never binds alongside $x + 2y \leq 20$ – it is redundant. Vertices and profits: $(0, 0)$: 0; $(10, 0)$ ($x + 2y = 20$): 400; $(8, 6)$ ($x + 2y = 20$, $y = 6$): $40(8) + 70(6) = 740$; $(0, 6)$: 420. Maximum $P = 740$ at $(8, 6)$. (c) $(8, 6)$ is already integral: 8 chairs and 6 tables ✓ (check neighbouring integers $(8, 5)$: $P = 320 + 350 = 670 < 740$; $(9, 5)$: violates $x + 2y \leq 20$? $9 + 10 = 19 \leq 20$ ✓ and $2(9) + 4(5) = 38 \leq 48$ ✓, $P = 360 + 350 = 710 < 740$ ✓).

Exercise 3.2

3.2 (*Original.*) Minimise the cost $C = 5x + 4y$ subject to $x + 2y \geq 8$, $2x + y \geq 10$, $x, y \geq 0$. (a) Sketch the feasible region (it is unbounded towards large x, y) and list its vertices. (b) Find the minimum by the vertex method. (c) Explain why an unbounded region does not prevent a minimum here. [7]

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3.2 (a) The region lies above both lines; vertices: $(0, 10)$ (on $2x + y = 10$), $(4, 2)$ (intersection of $x + 2y = 8$ and $2x + y = 10$: solving, $y = 2$, $x = 4$), and $(8, 0)$ (on $x + 2y = 8$); the region extends to infinity up-right. (b) C at the vertices: $(0, 10)$: 40; $(4, 2)$: 28; $(8, 0)$: 40. Minimum $C = 28$ at $(4, 2)$. (c) Increasing x or y only adds cost (coefficients positive), so the unbounded directions cannot lower C ; the minimum must occur at a vertex.

Local review guidance: every constraint captured with non-negativity; vertices found as constraint intersections, tested in the objective; integer checks at neighbouring lattice points inside the region. This is authored classroom guidance.