

Exercise walk-through

9.3

Route inspection and the travelling salesman problem

Pearson Edexcel · International A-Level · Mathematics

7. Critical path analysis

Critical paths

Precedence table to activity-on-arc network; dummies carry dependencies. Forward pass takes maxima (earliest event times), backward pass minima (latest). Float = latest finish – earliest start – duration; critical activities float zero. The example A(3), B(4), C(4), D(1), E(5) has critical path A–C–E = 12 days and D floats 2; Gantt bars run from earliest starts.

Check yourself

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Bubble and quick sorts pass by pass; first-fit, decreasing and full-bin placements; Kruskal and Prim (matrix form too); Dijkstra with permanence order and route recovery; odd-node pairings for the postman; TSP bounds both sides with nearest neighbour; dummies, forward and backward passes, floats, Gantt charts; LP formulation and vertex solutions with integer checks.

Exercises 2.1·2.2

2.1 A network has edges AB 5, BC 2, CD 4, DA 3, AC 6 with total 20. Find the odd nodes, the least duplication, and the length of the shortest closed route covering every edge. [4]

2.2 For a complete network on W, X, Y, Z with WX 3, WY 4, WZ 5, XY 2, XZ 6, YZ 7: find the nearest neighbour tour from W and its length. [3]

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2.1 Degrees: A 3 (AB, AD, AC), B 2, C 3 (BC, CD, CA), D 2 – odd nodes A and C. The only pairing is AC (6): route length $20 + 6 = 26$.

2.2 For a complete network on W, X, Y, Z with WX 3, WY 4, WZ 5, XY 2, XZ 6, YZ 7: find the nearest neighbour tour from W and its length. [3]

2.2 From W: nearest X (3); from X: nearest Y (2); from Y: nearest Z (7)? $YZ = 7$ vs $YW = 4$ but W is visited – from Y the unvisited are Z only: Y–Z (7); return Z–W (5). Tour W–X–Y–Z–W = $3 + 2 + 7 + 5 = 17$.

Exercises 2.3·3.1

2.3 The MST of a complete network (triangle inequality holding) weighs 12. State the upper bound obtained by doubling the MST, and explain in one sentence why shortcuts can only improve it. [2]

3.1 (*Original.*) A roads inspector must travel along each road of the network with edge weights AB 6, AC 4, BC 5, CD 2, BD 7, AD 3 at least once, starting and ending at A. (a) Identify the vertices of odd degree. (b) List the possible pairings of the odd nodes with their total weights and choose the least. (c) Find the length of the inspector's shortest route. (d) State, with a reason, whether the route needs to repeat any roads. [9]

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2.3 Doubled MST: 24. A closed walk along each MST edge twice visits every vertex but repeats a vertex whenever the tree branches; replacing such repeated stretches by direct edges can only shorten the tour because the triangle inequality makes the direct edge no longer than the two-leg path.

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3.1 (a) Degrees: A 3 (AB, AC, AD), B 3 (AB, BC, BD), C 3 (AC, BC, CD), D 3 (CD, BD, AD) – all four vertices are odd. (b) Pairings: $AB + CD = 6 + 2 = 8$; $AC + BD = 4 + 7 = 11$; $AD + BC = 3 + 5 = 8$. Least = 8 (either $AB + CD$ or $AD + BC$). (c) Total of all edges = $6 + 4 + 5 + 2 + 7 + 3 = 27$; shortest route = $27 + 8 = 35$. (d) Yes: four odd-degree vertices cannot form a closed trail using each edge once – exactly the chosen pairing's edges (AB and CD, or AD and BC) must be repeated.

Exercise 3.2

3.2 (*Original.*) A sales representative must visit each of four cities J, K, L, M once and return to the start. The complete table of shortest distances is JK 8, JL 5, JM 9, KL 4, KM 10, LM 6. (a) Use the nearest neighbour algorithm from J to find a tour and state its length. (b) Find the minimum spanning tree of the network and use it to obtain a lower bound for the optimal tour by deleting J. (c) Write down an interval containing the optimal tour length. [9]

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3.2 (a) From J: nearest L (5); from L: nearest K (4); from K: nearest M (10); return M–J (9). Tour J–L–K–M–J = $5 + 4 + 10 + 9 = 28$. (b) MST (Kruskal): KL 4 ✓, JL 5 ✓, LM 6 ✓ (tree on J, K, L, M), total 15. Deleting J: MST of {K, L, M} = KL 4 + LM 6 = 10, plus J's two cheapest edges JL 5 + JK 8 = 13: lower bound = $10 + 13 = 23$. (c) The optimal tour lies between 23 and 28 (the nearest-neighbour upper bound).

Local review guidance: odd nodes listed with degrees; ALL pairings costed before choosing; postman total = edges + chosen pairing; NN tour and the delete-a-vertex lower bound assembled from the MST of the rest plus the two cheapest at the deleted vertex. This is authored classroom guidance.