Exercise walk-through

4.5 Integration methods and volumes of revolution

Pearson Edexcel · International A-Level · Mathematics

Check yourself

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Contradiction proofs with the assumption named; partial fractions with repeated factors and improper cases; parametric conversions and gradients; the general binomial series with validity; implicit tangents; separable equations with conditions; parts, substitution and partial-fraction integrals; exact volumes of revolution; vector angles from the scalar product.

Exercises 2.1·2.3

2.1 Use the substitution $u = x^2 + 1$ to find $\int x(x^2+1)^3\,dx$. [3]

2.2 Find $\int x\mathrm{e}^{2x}\,dx$ by parts. [3]

2.3 Find $\int \ln 3x\,dx$. [3]

Exercises 2.1·2.3

2.1 Use the substitution $u = x^2 + 1$ to find $\displaystyle\int x(x^2+1)^3\,dx$. [3]

2.1 $du = 2x\,dx$: $\frac{1}{2}\int u^3 du = \frac{(x^2+1)^4}{8} + c$.

2.2 Find $\displaystyle\int x\mathrm{e}^{2x}\,dx$ by parts. [3]

2.2 $\frac{1}{2}x\mathrm{e}^{2x} - \frac{1}{4}\mathrm{e}^{2x} + c$ (parts with $u = x$, $\frac{dv}{dx} = \mathrm{e}^{2x}$).

2.3 Find $\displaystyle\int \ln 3x\,dx$. [3]

2.3 $x\ln 3x - x + c$ (parts with $u = \ln 3x$, $\frac{dv}{dx} = 1$).

Exercises 2.4·2.5

2.4 Find $\displaystyle\int \frac{4x+5}{(x+1)(x+2)}\,dx$ using partial fractions. [5]

2.5 Find the volume when $y = \sqrt{x}$ on $[0, 4]$ rotates about the x-axis. [4]

Exercises 2.4·2.5

2.4 Find $\displaystyle\int \frac{4x+5}{(x+1)(x+2)}\,dx$ using partial fractions. [5]

2.4 $\dfrac{4x+5}{(x+1)(x+2)} = \dfrac{1}{x+1} + \dfrac{3}{x+2}$ (check at $x=0$: LHS $\frac{5}{2}$, RHS $1+\frac{3}{2}$ ✓):

$$\ln|x+1| + 3\ln|x+2| + c$$

2.5 Find the volume when $y=\sqrt{x}$ on $[0,4]$ rotates about the x-axis. [4]

2.5 $V = \pi\displaystyle\int_0^4 x\,dx = \pi\left[\frac{x^2}{2}\right]_0^4 = 8\pi.$

Exercise 3.1

3.1 *(Adapted from Edexcel IAL WMA14 June 2025 Q7.)* Use the substitution $u = \tan x$ to find $\displaystyle\int \tan x \sec^2 x \, dx$. [4]

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3.1 $u = \tan x$: $\dfrac{du}{dx} = \sec^2 x$, so $\sec^2 x \, dx = du$:

$$\int \tan x \sec^2 x \, dx = \int u \, du = \frac{u^2}{2} + c = \frac{\tan^2 x}{2} + c$$

- Check: $\dfrac{d}{dx}\left(\frac{1}{2}\tan^2 x\right) = \tan x \sec^2 x$ ✓.

Exercise 3.2

3.2 *(Structure adapted from Edexcel IAL WMA14 June 2025 Q9.)* The region under $y = 2\sin x$ for $0 \leq x \leq \pi$ rotates about the x-axis. Find the exact volume, showing your use of a double-angle identity. [6]

Exercise 3.2

3.2 *(Structure adapted from Edexcel IAL WMA14 June 2025 Q9.)* The region under $y = 2\sin x$ for $0 \le x \le \pi$ rotates about the x-axis. Find the exact volume, showing your use of a double-angle identity. [6]

3.2 $V = \pi \int_0^{\pi} 4\sin^2 x\,dx = 4\pi \int_0^{\pi} \frac{1}{2}(1 - \cos 2x)\,dx = 2\pi \left[x - \frac{\sin 2x}{2}\right]_0^{\pi} = 2\pi^2.$

- Check: the mean of $\sin^2$ over a full half-turn is $\frac{1}{2}$: $4\pi \times \frac{1}{2} \times \pi = 2\pi^2$ ✓ exactly.

Local review guidance: substitution applied to dx too, or parts with the right u; exact antiderivatives and volumes; identity used before integrating a squared trig function. This is authored classroom guidance.