

Exercise walk-through

4.1

Proof by contradiction and partial fractions

Pearson Edexcel · International A-Level · Mathematics

Check yourself

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Contradiction proofs with the assumption named; partial fractions with repeated factors and improper cases; parametric conversions and gradients; the general binomial series with validity; implicit tangents; separable equations with conditions; parts, substitution and partial-fraction integrals; exact volumes of revolution; vector angles from the scalar product.

Exercises 2.1·2.3

2.1 Prove by contradiction that $\sqrt{3}$ is irrational. [4]

2.2 Decompose $\frac{7x + 10}{(x + 1)(x + 3)}$. [3]

2.3 Decompose $\frac{5x + 7}{(x - 1)^2}$. [3]

Exercises 2.1·2.3

2.1 Prove by contradiction that $\sqrt{3}$ is irrational. [4]

2.1 Suppose $\sqrt{3} = \frac{p}{q}$ in lowest terms. Then $3q^2 = p^2$, so p^2 is divisible by 3, so p is (if $p = 3m$, $p^2 = 9m^2$; conversely a prime dividing p^2 divides p). Write $p = 3m$: $3q^2 = 9m^2 \Rightarrow q^2 = 3m^2$, so q is divisible by 3 too – both divisible contradicts lowest terms. ■

2.2 Decompose $\frac{7x+10}{(x+1)(x+3)}$. [3]

2.2 $7x+10 = A(x+3) + B(x+1)$: at $x = -1$, $A = \frac{3}{2}$; at $x = -3$, $B = \frac{11}{2}$. (Or compare coefficients: $A+B=7$, $3A+B=10$.)

2.3 Decompose $\frac{5x+7}{(x-1)^2}$. [3]

2.3 With $\frac{5x+7}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$, multiply through: $5x+7 = A(x-1) + B$. Comparing x -coefficients, $A=5$; comparing constants, $7 = -5 + B \Rightarrow B = 12$.

Exercises 2.4·3.1

2.4 Write $\frac{x^2 + 2x + 5}{x^2 + x}$ in the form $1 + \frac{A}{x} + \frac{B}{x + 1}$. [4]

3.1 (*Adapted from Edexcel IAL WMA14 June 2025 Q10.*) Use proof by contradiction to show that $|\sin x + \cos x| < 1$ for all angles x with $90^\circ < x < 180^\circ$. [4]

Exercises 2.4·3.1

2.4 Write $\frac{x^2 + 2x + 5}{x^2 + x}$ in the form $1 + \frac{A}{x} + \frac{B}{x + 1}$. [4]

2.4 $\frac{x^2 + 2x + 5}{x^2 + x} = 1 + \frac{x + 5}{x^2 + x} = 1 + \frac{x + 5}{x(x + 1)}$; decompose $\frac{x + 5}{x(x + 1)} = \frac{5}{x} - \frac{4}{x + 1}$. So $A = 5$, $B = -4$. Check at $x = 1$: $1 + 5 - 2 = 4$; original $\frac{8}{2} = 4$ ✓.

3.1 (*Adapted from Edexcel IAL WMA14 June 2025 Q10.*) Use proof by contradiction to show that $|\sin x + \cos x| < 1$ for all angles x with $90^\circ < x < 180^\circ$. [4]

3.1 Suppose some x in $90^\circ < x < 180^\circ$ has $|\sin x + \cos x| \geq 1$. Squaring: $1 + \sin 2x \geq 1$, so $\sin 2x \geq 0$. But $180^\circ < 2x < 360^\circ$ forces $\sin 2x < 0$ – a contradiction. So $|\sin x + \cos x| < 1$ throughout. ■ Check at $x = 135^\circ$: $|0| = 0 < 1$ ✓.

Exercise 3.2

3.2 Decompose $\frac{2x^2 + 5x + 4}{(x + 2)(x + 1)^2}$ and verify your decomposition at $x = 1$. [5]

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$$\mathbf{3.2} \quad \frac{2x^2 + 5x + 4}{(x + 2)(x + 1)^2} = \frac{A}{x + 2} + \frac{B}{x + 1} + \frac{C}{(x + 1)^2}: 2x^2 + 5x + 4 = A(x + 1)^2 + B(x + 2)(x + 1) + C(x + 2).$$

- $x = -2$: $8 - 10 + 4 = 2 = A$. $x = -1$: $2 - 5 + 4 = 1 = C$. x^2 : $2 = A + B \Rightarrow B = 0$.

$$\frac{2}{x + 2} + \frac{1}{(x + 1)^2}$$

Check at $x = 1$: LHS = $\frac{2+5+4}{(1+2)(1+1)^2} = \frac{11}{12}$; RHS = $\frac{2}{3} + \frac{1}{4} = \frac{11}{12}$ ✓.

Local review guidance: assumption of the negation stated, or correct partial-fraction identity set up; exact values; verification or the named contradiction. This is authored classroom guidance.