Exercise walk-through

3.4 Exponentials, logarithms and log-graph models

Pearson Edexcel · International A-Level · Mathematics

Check yourself

Check yourself

Rational simplification and inverses verified by composing; modulus cases checked; combined transformations; sec, cosec, cot with identities; harmonic form with exact R; compound and double angle proofs; log-graph recovery; growth-decay models with floor; product, quotient, chain and implicit differentiation; recognition integrals; sign-change and iteration methods.

Exercises 2.1·2.3

2.1 Solve $e^{2x} = 50$, giving x to 3 significant figures. [2]

2.2 Solve $\ln(3x - 2) = 4$ exactly. [2]

2.3 A population is modelled by $P = 2000e^{0.05t}$. Find P when $t = 10$ and state the model's initial value. [2]

Exercises 2.1·2.3

2.1 Solve $e^{2x} = 50$, giving x to 3 significant figures. [2]

2.1 $x = \frac{1}{2}\ln 50 = 1.96$ (3 s.f.).

2.2 Solve $\ln(3x - 2) = 4$ exactly. [2]

2.2 $3x - 2 = e^4$, so $x = \dfrac{e^4 + 2}{3}$.

2.3 A population is modelled by $P = 2000e^{0.05t}$. Find P when $t = 10$ and state the model's initial value. [2]

2.3 $P = 2000e^{0.5} = 3297$; initial value 2000 (at $t = 0$).

Exercises 2.4·3.1

2.4 For $y = ax^n$, a $\log y$–$\log x$ plot has gradient 3 and intercept 0.778 ($= \log 6$). Find a and n. [2]

2.5 Write $\log_{10} y = 2x + 1$ as an exponential relationship between y and x. [2]

3.1 *(Adapted from Edexcel IAL WMA13 June 2025 Q3.)* A $\log_{10} y$ against $\log_{10} x$ line passes through $(-2, 1)$ and $(0, 5)$. Express y in the form px^q, giving p and q exactly. [5]

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2.4 $n = 3$ and $\log a = \log 6 \Rightarrow a = 6$: $y = 6x^3$.

2.5 Write $\log_{10} y = 2x + 1$ as an exponential relationship between y and x. [2]

2.5 $y = 10^{2x+1} = 10 \times 100^x$.

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3.1 Gradient $= \dfrac{5-1}{0-(-2)} = 2$; intercept 5:

$$\log_{10} y = 2\log_{10} x + 5 \implies y = 10^5\, x^2 = 100\,000\, x^2$$

$p = 100\,000$, $q = 2$. Check at $(0, 5)$: $x = 1$, $y = 10^5$, $\log y = 5$ ✓.

Exercise 3.2

3.2 *(Adapted from Edexcel IAL WMA13 June 2025 Q6.)* The temperature of an engine is modelled by $\theta = 25 + Ae^{-kt}$ (θ in °C, t in minutes). The engine is 95 °C when switched off at $t = 0$, and 40 °C at $t = 6$.

(a) Find A and k (3 s.f.).

(b) Find the time at which the temperature is decreasing at 4 °C per minute (2 d.p.).

(c) Explain why the model cannot hold below 25 °C. [8]

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(c) Explain why the model cannot hold below 25 °C. [8]

3.2 (a) At $t = 0$: $95 = 25 + A \Rightarrow A = 70$. At $t = 6$: $40 = 25 + 70\mathrm{e}^{-6k} \Rightarrow \mathrm{e}^{-6k} = \frac{15}{70} = \frac{3}{14}$, so

$$k = \frac{1}{6}\ln\frac{14}{3} = 0.257 \text{ (3 s.f.)}$$

(b) $\dfrac{d\theta}{dt} = -70k\mathrm{e}^{-kt} = -4$:

Full solution on next slide

Full solution 3.2

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(b) $\dfrac{d\theta}{dt} = -70k\mathrm{e}^{-kt} = -4$:

$$\mathrm{e}^{-kt} = \frac{4}{70 \times 0.257} = 0.2226 \implies t = \frac{\ln(1/0.2226)}{0.257} = 5.85 \text{ (2 d.p.)}$$

Check: $\theta(5.85) = 25 + 70\mathrm{e}^{-0.257(5.85)} = 25 + 70(0.2226) = 40.6$ °C, falling at $70 \times 0.257 \times 0.2226 = 4.0$ °C/min ✓. (c) As $t \to \infty$, $\theta \to 25$ °C from above: the model's floor is room temperature, so it never predicts values below 25 °C – a colder-than-room prediction needs a new model.

Local review guidance: correct log recovery or correct differentiation of the model; exact/rounded values as asked; model interpretation tied to the limit. This is authored classroom guidance.