Exercise walk-through

3.3 Reciprocal trig and harmonic form

Pearson Edexcel · International A-Level · Mathematics

Check yourself

Check yourself

Rational simplification and inverses verified by composing; modulus cases checked; combined transformations; sec, cosec, cot with identities; harmonic form with exact R; compound and double angle proofs; log-graph recovery; growth-decay models with floor; product, quotient, chain and implicit differentiation; recognition integrals; sign-change and iteration methods.

Exercises 2.1·2.3

2.1 Write $\sec\theta$ in terms of $\cos\theta$ and state the range of $\sec\theta$. [2]

2.2 Simplify $\cot\theta\tan\theta$. [1]

2.3 State the restricted domain and range of arccos. [2]

Exercises 2.1·2.3

2.1 Write $\sec\theta$ in terms of $\cos\theta$ and state the range of $\sec\theta$. [2]

2.1 $\sec\theta = \dfrac{1}{\cos\theta}$; since $|\cos\theta| \leq 1$, $\sec\theta \geq 1$ or $\sec\theta \leq -1$.

2.2 Simplify $\cot\theta\tan\theta$. [1]

2.2 $\cot\theta\tan\theta = 1$ (where both are defined).

2.3 State the restricted domain and range of arccos. [2]

2.3 arccos maps $[-1, 1]$ onto $[0, \pi]$.

Exercises 2.4·3.1

2.4 Use $\sin 2A \equiv 2\sin A\cos A$ to write $\sin A\cos A$ as a single term. [2]

2.5 Express $12\cos\theta + 5\sin\theta$ in the form $R\cos(\theta - \alpha)$ with exact R and $\alpha = \arctan\frac{5}{12}$. [3]

3.1 *(Adapted from Edexcel IAL WMA13 June 2025 Q2.)* Express $8\cos x - 15\sin x$ in the form $R\cos(x + \alpha)$, $R > 0$, $0 < \alpha < \frac{\pi}{2}$, giving R exactly and α to 3 decimal places. Hence write down the maximum value of $8\cos x - 15\sin x$ and the smallest positive x at which it occurs. [6]

Exercises 2.4·3.1

2.4 Use $\sin 2A \equiv 2\sin A\cos A$ to write $\sin A\cos A$ as a single term. [2]

2.4 $\sin A\cos A = \frac{1}{2}\sin 2A$.

2.5 Express $12\cos\theta + 5\sin\theta$ in the form $R\cos(\theta - \alpha)$ with exact R and $\alpha = \arctan\frac{5}{12}$. [3]

2.5 $R = \sqrt{144+25} = 13$, $\alpha = \arctan\frac{5}{12}$: $12\cos\theta + 5\sin\theta = 13\cos(\theta - \alpha)$. Check: $13\cos\alpha = 12$ ✓ and $13\sin\alpha = 5$ ✓ (5-12-13 triangle).

3.1 *(Adapted from Edexcel IAL WMA13 June 2025 Q2.)* Express $8\cos x - 15\sin x$ in the form $R\cos(x+\alpha)$, $R > 0$, $0 < \alpha < \frac{\pi}{2}$, giving R exactly and α to 3 decimal places. Hence write down the maximum value of $8\cos x - 15\sin x$ and the smallest positive x at which it occurs. [6]

3.1 $R = \sqrt{64+225} = 17$; $17\cos\alpha = 8$, $17\sin\alpha = 15$ so $\tan\alpha = \frac{15}{8}$, $\alpha = 1.080$ (3 d.p.):

$$8\cos x - 15\sin x = 17\cos(x + 1.080)$$

Maximum $= 17$ when $x + 1.080 = 0 \pmod{2\pi}$; the smallest positive x is $2\pi - 1.080 = 5.203$ (3 d.p.). Check: $\cos(5.203 + 1.080) = \cos(6.283) = \cos 2\pi = 1$ ✓.

Exercise 3.2

3.2 *(Adapted from Edexcel IAL WMA13 June 2025 Q8a.)* Prove, using the compound angle formulae, that

$$\tan 3x \equiv \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

[4]

Exercise 3.2

3.2 *(Adapted from Edexcel IAL WMA13 June 2025 Q8a.)* Prove, using the compound angle formulae, that

$$\tan 3x \equiv \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

[4]

3.2 Write $3x = 2x + x$:

$$\tan 3x = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}, \qquad \tan 2x = \frac{2t}{1 - t^2} \quad (t = \tan x)$$

Substitute and multiply through by $(1 - t^2)$:

$$\tan 3x = \frac{\frac{2t}{1-t^2} + t}{1 - \frac{2t^2}{1-t^2}} = \frac{2t + t - t^3}{(1 - t^2) - 2t^2} = \frac{3t - t^3}{1 - 3t^2} \qquad \blacksquare$$

Local review guidance: correct R and matching of both components; exact or correctly rounded values; maximum read from the amplitude with its location. This is authored classroom guidance.