

Exercise walk-through

2.9

Definite integrals and areas

Pearson Edexcel · International A-Level · Mathematics

Check yourself

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Exhaustion with complete cases; counterexamples; factor/remainder theorems with $(ax - b)$; circles from any form plus the three properties; both series families with proofs and logs; binomial coefficients; log laws with domain checks; identity-reduced trig equations in either unit; stationary points classified and modelled; areas by definite integral and by the trapezium rule with the error direction.

Exercises 2.1·2.3

2.1 Evaluate $\int_1^3 2x \, dx$. [2]

2.2 Evaluate $\int_0^4 \sqrt{x} \, dx$. [3]

2.3 Evaluate $\int_0^{\pi/2} \cos x \, dx$. [2]

Exercises 2.1·2.3

2.1 Evaluate $\int_1^3 2x \, dx$. [2]

2.1 $[x^2]_1^3 = 9 - 1 = 8$.

2.2 Evaluate $\int_0^4 \sqrt{x} \, dx$. [3]

2.2 $\left[\frac{2}{3}x^{3/2}\right]_0^4 = \frac{2}{3}(8) = \frac{16}{3}$.

2.3 Evaluate $\int_0^{\pi/2} \cos x \, dx$. [2]

2.3 $[\sin x]_0^{\pi/2} = 1 - 0 = 1$.

Exercises 2.4·2.5

2.4 Find the area under $y = x^2$ from $x = 0$ to $x = 3$. [2]

2.5 Find the area between $y = \sqrt{x}$, $y = x$ and the lines $x = 0$, $x = 1$. [4]

Exercises 2.4·2.5

2.4 Find the area under $y = x^2$ from $x = 0$ to $x = 3$. [2]

$$\mathbf{2.4} \quad \left[\frac{x^3}{3} \right]_0^3 = 9.$$

2.5 Find the area between $y = \sqrt{x}$, $y = x$ and the lines $x = 0$, $x = 1$. [4]

$$\mathbf{2.5} \quad \sqrt{x} \geq x \text{ on } [0, 1]: \text{ area} = \int_0^1 (\sqrt{x} - x) dx = \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2} \right]_0^1 = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}.$$

Exercise 3.1

3.1 (*Adapted from Edexcel IAL WMA12 June 2025 Q3.*) The curve C has equation $y = 2x + \frac{3}{x} - 4$ ($x > 0$), and the points $P(1, 1)$ and $Q(3, 3)$ lie on the line $y = x$. Find the area of the region bounded by C and the line segment PQ , giving your answer in exact form. [6]

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3.1 First confirm P, Q lie on C : $C(1) = 2 + 3 - 4 = 1$ ✓; $C(3) = 6 + 1 - 4 = 3$ ✓. At $x = 2$ (between): $C = 4 + 1.5 - 4 = 1.5$ and the line gives 2, so the line is above:

$$\begin{aligned} \text{Area} &= \int_1^3 \left[x - \left(2x + \frac{3}{x} - 4 \right) \right] dx = \int_1^3 \left(-x - \frac{3}{x} + 4 \right) dx \\ &= \left[-\frac{x^2}{2} - 3 \ln x + 4x \right]_1^3 = \left(-\frac{9}{2} - 3 \ln 3 + 12 \right) - \left(-\frac{1}{2} + 4 \right) = \frac{15}{2} - 3 \ln 3 - \frac{7}{2} \\ &= 4 - 3 \ln 3 \end{aligned}$$

Check: $3 \ln 3 \approx 3.30$, so the area is $4 - 3.30 = 0.70$ – positive ✓ (integrating the right way round).

Exercise 3.2

3.2 Find the area of the region bounded by the curve $y = x^2 - 4x + 5$ and the line $y = 5$. [5]

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3.2 Intersections: $x^2 - 4x + 5 = 5 \implies x(x - 4) = 0 \implies x = 0, 4$; the line is on top (at $x = 2$: curve 1, line 5).

$$\text{Area} = \int_0^4 [5 - (x^2 - 4x + 5)] dx = \int_0^4 (-x^2 + 4x) dx = \left[-\frac{x^3}{3} + 2x^2 \right]_0^4 = -\frac{64}{3} + 32 = \frac{32}{3}$$

Check: $32/3 \approx 10.7$ – a cap 4 wide and up to 4 deep averages about 2.7 ✓.

Local review guidance: correct antiderivative and limits from the intersections; exact value; integrand ordered upper-minus-lower from a sketch check. This is authored classroom guidance.