Exercise walk-through

2.6
Exponentials and logarithms

Pearson Edexcel · International A-Level · Mathematics

Check yourself

Check yourself

Exhaustion with complete cases; counterexamples; factor/remainder theorems with $(ax - b)$; circles from any form plus the three properties; both series families with proofs and logs; binomial coefficients; log laws with domain checks; identity-reduced trig equations in either unit; stationary points classified and modelled; areas by definite integral and by the trapezium rule with the error direction.

Exercises 2.1·2.3

2.1 Solve $5^x = 125$. [2]

2.2 Solve $4^{x-1} = 8$ by writing both sides as powers of 2. [3]

2.3 Write $2\log_{10} 3 + \log_{10} 8 - \log_{10} 6$ as a single logarithm. [3]

Exercises 2.1·2.3

2.1 Solve $5^x = 125$. [2]

2.1 $5^x = 5^3$, so $x = 3$.

2.2 Solve $4^{x-1} = 8$ by writing both sides as powers of 2. [3]

2.2 $4^{x-1} = 2^{2x-2}$ and $8 = 2^3$: $2x - 2 = 3$, so $x = \frac{5}{2}$.

2.3 Write $2\log_{10} 3 + \log_{10} 8 - \log_{10} 6$ as a single logarithm. [3]

2.3 $\log_{10}\left(\dfrac{3^2 \times 8}{6}\right) = \log_{10} 12$.

Exercises 2.4·2.5

2.4 Solve $\log_2(x+6) - \log_2 x = 1$. [3]

2.5 Describe the key features of the graph of $y = 2^x$. [2]

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2.4 Solve $\log_2(x+6) - \log_2 x = 1$. [3]

2.4 $\log_2 \dfrac{x+6}{x} = 1 \implies \dfrac{x+6}{x} = 2 \implies x+6 = 2x \implies x = 6$. Domain: both $x+6$ and x positive ✓. Check: $\log_2 12 - \log_2 6 = \log_2 2 = 1$ ✓.

2.5 Describe the key features of the graph of $y = 2^x$. [2]

2.5 Passes through $(0,1)$; increases for all x; the x-axis ($y = 0$) is an asymptote as $x \to -\infty$.

Exercise 3.1

3.1 *(Adapted from Edexcel IAL WMA12 June 2025 Q6(i).)* Solve $2\log_2(x-4) = \log_2(x+8)$. [4]

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3.1 $2\log_2(x-4) = \log_2(x-4)^2 = \log_2(x+8)$, so

$$(x-4)^2 = x+8 \implies x^2 - 8x + 16 = x + 8 \implies x^2 - 9x + 8 = 0$$

$$(x-1)(x-8) = 0 \implies x = 1 \text{ or } x = 8$$

Domain: $x - 4 > 0$ needs $x > 4$, so $x = 1$ is rejected and $x = 8$. Check: $2\log_2 4 = 4$ and $\log_2 16 = 4$ ✓.

Exercise 3.2

3.2 *(Adapted from the same question.)* The curves $y = 3^{2x}$ and $y = 4 \times 3^x$ meet at the point P. Find the exact coordinates of P, giving your answer in the form $(\log_3 a,\ b)$ where a and b are integers. [5]

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3.2 Set the expressions equal and substitute $u = 3^x$:

$$3^{2x} = 4 \times 3^x \implies u^2 = 4u \implies u = 4\ (u \neq 0) \implies 3^x = 4 \implies x = \log_3 4$$

$$y = 3^{2x} = (3^x)^2 = 4^2 = 16$$

$P = (\log_3 4,\ 16)$: $a = 4$, $b = 16$. Check on the other curve: $4 \times 3^{\log_3 4} = 4 \times 4 = 16$ ✓.

Local review guidance: correct log law applied; quadratic solved exactly; domain rejection stated and the surviving root checked. This is authored classroom guidance.