# Pure Mathematics 1

Edexcel IAL P1 (WMA11/01); algebra, coordinate geometry, trigonometry, differentiation, integration

# Pure Mathematics 1: the toolkit every later unit uses

- work with indices and surds, quadratics, **simultaneous equations** and inequalities;
- sketch curves and describe transformations of graphs;
- find equations of straight lines, including **parallel** and **perpendicular** lines;

# Pure Mathematics 1: the toolkit every later unit uses (2)

- use the **sine rule**, the **cosine rule** and radians;
- differentiate and integrate powers of $x$.

# Algebra and functions

- Known: $p = x^4/16$, and a power $\frac{1}{2}$ means square root.
- Check: squaring the answer, $(x^2/4)^2 = x^4/16 = p$.
- Known: multiply before inverting, using $x^4 \cdot x^{-1} = x^3$.

$$a^m \times a^n = a^{m+n} \qquad a^m \div a^n = a^{m-n} \qquad (a^m)^n = a^{mn}$$

# Algebra and functions (2)

- Check: $(pq) \times (pq)^{-1} = \frac{5x^3}{6} \times \frac{6}{5x^3} = 1$.
- Check: $5^2 = 25$, $6^2 = 36$, $(x^3)^2 = x^6$, and the constants are already fully simplified.
- Known: $(1+\sqrt{2})(\sqrt{2}-1) = \sqrt{2} - 1 + 2 - \sqrt{2} = 1$, so multiply by $\frac{\sqrt{2}-1}{\sqrt{2}-1}$.

$$p^{\frac{1}{2}} = \sqrt{\frac{x^4}{16}} = \frac{\sqrt{x^4}}{\sqrt{16}} = \frac{x^2}{4}$$

# Algebra and functions (3)

- Check by **calculator**: $12/(1 + 1.4142\ldots) = 4.9706\ldots$ and $12\sqrt{2} - 12 = 4.9706\ldots$ ✓
- $\Delta > 0$: two distinct real roots — the curve cuts the $x$-axis twice;
- $\Delta = 0$: one repeated root — the curve touches the $x$-axis;

$$pq = \frac{x^4}{16} \times \frac{40}{3x} = \frac{40x^3}{48} = \frac{5x^3}{6}$$

# Algebra and functions (4)

- $\Delta < 0$: no real roots — the curve never meets the $x$-axis.
- Known: with $k = 4.5$ the equation is $3.5x^6 + 4x^3 + 0.5 = 0$; multiply by 2 to clear decimals:
- Why substitute: $x^6 = (x^3)^2$, so this is a quadratic in $x^3$. Substitute $z = x^3$:

$$(pq)^{-1} = \left(\frac{5x^3}{6}\right)^{-1} = \frac{6}{5}x^{-3}$$

# Algebra and functions (5)

- Return to $x$: $x^3 = -\frac{1}{7}$ or $x^3 = -1$, so
- Check: $z = -1$ gives $7 - 8 + 1 = 0$ ✓; $z = -\frac{1}{7}$ gives $7 \cdot \frac{1}{49} - \frac{8}{7} + 1 = \frac{1}{7} - \frac{8}{7} + 1 = 0$ ✓
- Known: treat the equation as a quadratic in $x^3$ with $a = k - 1$, $b = 4$, $c = k - 4$. No real

$$(pq)^2 = \left(\frac{5x^3}{6}\right)^2 = \frac{25x^6}{36}$$

# Algebra and functions (6)

- Check: at $k = -1$: $4(-1)(6) = -24 < 0$ ✓; at $k = 6$: $4(6)(-1) = -24 < 0$ ✓; at $k = 3$:
- Expand both sides: $0.2x^3 - 0.8x^2 = 9.6x - 1.2x^2$.
- Collect terms: $0.2x^3 + 0.4x^2 - 9.6x = 0$, then divide by $0.2$: $x^3 + 2x^2 - 48x = 0$.

$$\frac{12}{1+\sqrt{2}} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{12(\sqrt{2}-1)}{(1+\sqrt{2})(\sqrt{2}-1)} = 12(\sqrt{2}-1) = 12\sqrt{2} - 12$$

# Algebra and functions (7)

- Factorise: $x(x^2 + 2x - 48) = x(x+8)(x-6) = 0$, so $x = 0, -8, 6$.
- Substitute back into $C_2$: $(0, 0)$, $(-8, -153.6)$, $(6, 14.4)$.
- Check with $C_1$: $0.2(-8)^2(-12) = 0.2 \cdot 64 \cdot (-12) = -153.6$ ✓ and $0.2(36)(2) = 14.4$ ✓

$$7x^6 + 8x^3 + 1 = 0$$

# Algebra and functions (8)

- Solve $ax + b > cx + d$ by collecting $x$ (the **inequality** sign survives adding/subtracting, and survives dividing by a **positive** number; dividing by a negative flips the sign).
- Solve $px^2 + qx + r \leq 0$ by sketching: the solution is where the parabola is on or below the axis: between two roots when $p > 0$, and outside them when $p < 0$. Check repeated/no-real-root cases separately.

$$7z^2 + 8z + 1 = 0 \implies (7z + 1)(z + 1) = 0 \implies z = -\tfrac{1}{7} \text{ or } z = -1$$

# Algebra and functions (9)

- "Curve below line" inequalities such as $px^2 + qx + r < ax + b$ mean: where is the curve's height less than the line's height? Find intersections first.
- When **sketching** the solution region, use a **dotted line** for strict $<$ or $>$ boundaries and a **solid line** for $\leq$ or $\geq$.
- Rearrange: $x^2 - 3x - 10 < 0$, and factorise: $(x-5)(x+2) < 0$.

$$x = -\sqrt[3]{\tfrac{1}{7}} = -\frac{1}{\sqrt[3]{7}} = -\frac{\sqrt[3]{49}}{7} \qquad \text{or} \qquad x = -1$$

# Algebra and functions (10)

- The parabola $y = (x-5)(x+2)$ opens upward, so it is below zero **between** its roots:
- Check: $x = 0$ gives $0 < 10$ ✓; $x = 6$ gives $36 < 28$ × (correctly excluded).
- $y = \sqrt{2x}$ from $y = \sqrt{x}$: write $\sqrt{2x} = f(2x)$, so this is a **stretch parallel to the $x$-axis with scale factor $\frac{1}{2}$**.

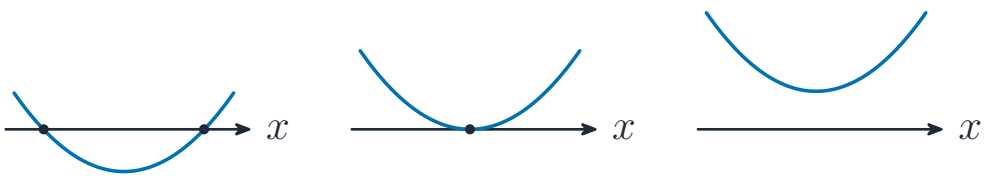
$$b^2 - 4ac = 16 - 4(k-1)(k-4) = 16 - 4(k^2 - 5k + 4) = -4k^2 + 20k = 4k(5-k)$$

# Algebra and functions (11)

- $y = 12 - \sqrt{x}$ from $y = -\sqrt{x}$: write $12 - \sqrt{x} = -\sqrt{x} + 12$, a **translation by** $\begin{pmatrix} 0 \\ 12 \end{pmatrix}$ (up by 12).
- Check with a point: $y = \sqrt{x}$ passes $(4, 2)$; after the first **transformation** the curve passes $(2, 2)$ (halved $x$), and $y = \sqrt{2 \times 2} = 2$ ✓. After the second, $y = -\sqrt{4} + 12 = 10$, so $(4, 10)$ lies on it ✓.

$$4k(5 - k) < 0 \implies k < 0 \text{ or } k > 5$$

# Figure: Algebra and functions



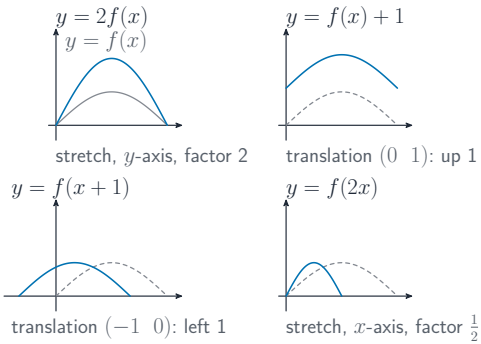

$b^2 - 4ac > 0$
two distinct roots

$b^2 - 4ac = 0$
one repeated root

$b^2 - 4ac < 0$
no real roots

- The **discriminant** counts intersections with the x-axis.
- Read the axes and units first.

# Figure: Algebra and functions (2)




- Name the direction, the type and the scale factor (or vector).
- Read the axes and units first.

# Coordinate geometry in the (x, y) plane

- **Parallel** lines have the same gradient: $m_1 = m_2$.
- **Perpendicular** lines have gradients that multiply to $-1$: $m_1 m_2 = -1$. (A horizontal line and a vertical line are also **perpendicular**.)
- Known: rearrange $l_1$: $2y = x + 25$, so $y = \frac{1}{2}x + \frac{25}{2}$ and $m_1 = \frac{1}{2}$.

$$y - 0 = -2(x - 0) \implies y = -2x$$

# Coordinate geometry in the (x, y) plane (2)

- Why: **perpendicular** gradients multiply to $-1$, so $m_2 = -1 \div \frac{1}{2} = -2$.
- Substitute $y = -2x$ into $x - 2y + 25 = 0$:
- $P = (-5, 10)$. Check in $l_1$: $-5 - 20 + 25 = 0$ ✓

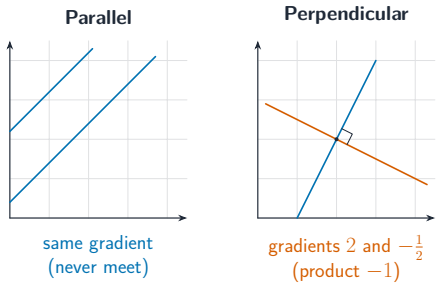
$$x - 2(-2x) + 25 = 0 \implies 5x + 25 = 0 \implies x = -5, \quad y = -2(-5) = 10$$

# Coordinate geometry in the (x, y) plane (3)

- The shortest distance is $|OP|$ along the **perpendicular**:
- Check: $\sqrt{125} = \sqrt{25 \times 5} = 5\sqrt{5} \approx 11.18$ ✓

$$|OP| = \sqrt{(-5-0)^2 + (10-0)^2} = \sqrt{25+100} = \sqrt{125} = 5\sqrt{5}$$

# Figure: Coordinate geometry in the (x, y) plane




- **Parallel** lines have equal gradients; **perpendicular** lines have gradients that multiply to $-1$.
- Match each part of the diagram to the text.

# Trigonometry

- Known: $OC = OB + BC = 15 + 2 = 17$ m; in triangle $AOC$, side $AO = 15$ is opposite the given angle $0.6$, and side $OC = 17$ is opposite angle $CAO$.
- **Sine rule** (quote it first):
- Check: angles sum to $\pi$: $0.6 + 0.6944 + 1.8471 \approx 3.1416$ ✓

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc\cos A \qquad \text{area} = \tfrac{1}{2}ab\sin C$$

# Trigonometry (2)

- Major sector angle: $2\pi - 1.847 = 4.436$ radians.
- Check: $\sin 1.847 = \sin(\pi - 1.847) = \sin 1.2946 = 0.962$ ✓
- The boundary is the major arc $AB$ plus sides $AC$ and $BC$.

$$\text{arc length } s = r\theta \qquad \text{sector area} = \tfrac{1}{2}r^2\theta$$

# Trigonometry (3)

- **Cosine rule** for $AC$ (quote it first):
- Check: $\cos 1.847 < 0$ (the angle is obtuse), so the $-2bc\cos A$ term adds ✓
- Rearrange: $4\cos x = -4$, so $\cos x = -1$, giving $x = 180° + 360°n$.

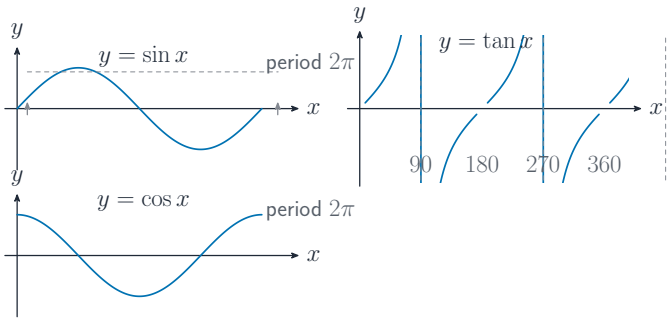
$$\frac{\sin CAO}{17} = \frac{\sin 0.6}{15} \implies \sin CAO = \frac{17\sin 0.6}{15} = 0.6399\ldots \implies CAO = 0.6944\ldots$$

# Trigonometry (4)

- In $-720° < x \leq 720°$: $x = -540°, -180°, 180°, 540°$ — **4 solutions**.
- Check: $\cos(\pm 180°) = -1$ ✓, and $5 + 4(-1) = 1$ ✓. The interval is half-open, so $x = -720°$ (where $\cos = 1$) is excluded correctly.

$$COA = \pi - 0.6 - 0.6944\ldots = 1.8471\ldots = 1.847\ (3\,\text{d.p.})$$

# Figure: Trigonometry




- Count periods first, then solutions per period.
- Read the axes and units first.

# Differentiation

- Differentiate (powers drop by one): $\dfrac{dy}{dx} = 2 \cdot \frac{5}{2}x^{3/2} - 4 = 5x^{3/2} - 4.$
- Set the gradient to 16 at $x = 2^k$:
- Check: $2^{4/3} = 2.5198\ldots$ and $5(2.5198)^{3/2} - 4 = 5(4.0) - 4 = 16$ ✓

$$y = (2x+5)(x-1) = 2x^2+3x-5 \implies \frac{dy}{dx} = 4x+3 \qquad y = \frac{3x^2+5}{x^2} = 3+5x^{-2} \implies \frac{dy}{dx} = -10$$

# Differentiation (2)

- Known: the tangent's gradient equals $f'(4)$:
- The tangent passes through $(4, 12)$: $12 = 10(4) + c \implies c = -28$.
- Check: $f'(4) = 6 + 16 \cdot \frac{1}{4} = 10$ ✓ and $10(4) - 28 = 12$ ✓
- Check: $\frac{3}{2\sqrt{4}} = \frac{3}{4}$ ✓

$$5(2^k)^{3/2} - 4 = 16 \implies (2^k)^{3/2} = 4 = 2^2 \implies 2^{3k/2} = 2^2 \implies \frac{3k}{2} = 2 \implies k = \frac{4}{3}$$

# Integration

- Expand the numerator: $(x+3)^2 = x^2 + 6x + 9$; the denominator is $3x^{1/2}$.
- Divide term by term (powers of $x$ subtract):
- Integrate each term (power up one, divide by the new power):

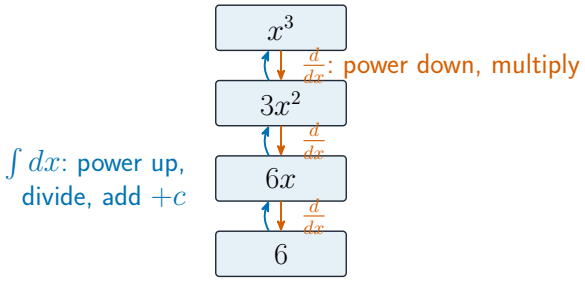
$$y = \int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

# Integration (2)

- Check by differentiating: $\frac{2}{15} \cdot \frac{5}{2}x^{3/2} = \frac{1}{3}x^{3/2}$ ✓, $\frac{4}{3} \cdot \frac{3}{2} = 2$ ✓, $6 \cdot \frac{1}{2} = 3$ ✓ — the integrand returns exactly.
- Substitute $P$: $2(4)^{3/2} + \frac{64}{12} + d = 12$, and $4^{3/2} = 8$:
- Check: $f(4) = 16 + \frac{16}{3} - \frac{28}{3} = 16 - 4 = 12$ ✓, and differentiating the answer gives $f'(x) = 3x^{1/2} + \frac{x^2}{4}$, so $f'(4) = 6 + 4 = 10$ ✓ (the tangent gradient from before).

$$\frac{x^2}{3x^{1/2}} = \frac{1}{3}x^{3/2} \qquad \frac{6x}{3x^{1/2}} = 2x^{1/2} \qquad \frac{9}{3x^{1/2}} = 3x^{-1/2}$$

# Figure: Integration




Each integral climbs one rung and picks up an arbitrary constant: $\int 6\,dx = 6x + c$, not just $6x$.

- Differentiation slides down; **integration** climbs back up, plus c.
- Match each part of the diagram to the text.

# Sources for this lesson

- Pearson Edexcel, *International Advanced Level Mathematics, Further Mathematics and Pure Mathematics specification* (Issue 3, 2019), unit P1.
- G. W. Leibniz, "Nova methodus pro maximis et minimis...", *Acta Eruditorum* (Leipzig, October 1684).
- Pearson Edexcel, WMA11/01 June 2025, questions 1 to 10 (adapted).

# Common errors

- Writing $a^{-1} = -a$; a negative power means a reciprocal, $a^{-1} = \frac{1}{a}$.
- Rationalising a denominator by multiplying the denominator only, not the numerator as well.
- Dividing both sides of an **inequality** by a negative number without reversing the sign.

# Common errors (2)

- Giving the solution of a quadratic **inequality** on the wrong side of the roots; sketch the parabola first.
- Moving a graph the wrong way: $y = f(x + a)$ moves it $a$ to the left, not the right.
- Using an angle in degrees in $s = r\theta$ or $\frac{1}{2}r^2\theta$.

# Common errors (3)

- Forgetting the second possible angle in the **ambiguous case** of the **sine rule**.
- Forgetting the **constant of integration** $+c$.

# Key terms

- **Surd**: an exact root such as $\sqrt{2}$ that is not a whole number.
- **Discriminant**: $b^2 - 4ac$, which shows how many real roots a quadratic has.
- **Asymptote**: a line that a curve gets closer and closer to.

# Key terms (2)

- **Radian**: the angle at the centre of a circle made by an arc equal in length to the radius.
- **Derivative**: the gradient function $\frac{dy}{dx}$ of a curve.
- **Constant of integration**: the constant $c$ added when integrating, because every vertical translation has the same **derivative**.

# Check yourself

- 1.
- Simplify $(8x^6)^{\frac{2}{3}}$. 2.
- Find the values of $k$ for which $x^2 + kx + 9 = 0$ has equal roots. 3.
- Solve $x^2 - x - 12 > 0$. 4.