8.2 Continuous random variables

Name: _____ Class: _____ Date: _____ **Total: 26 marks**

KEY WORDS probability density function • mode

Continuous random variables

Objective

Normalise piecewise densities, move between f and F in both directions, find medians, quartiles and modes, compute E(X), E(X^2) and Var, and read skewness from mean–median–mode (objectives 2.1–2.5).

1 Key knowledge and worked examples

Calculus kit. $\int f = 1$ pins constants; $f = F'$; $F(x) = \int_{-\infty}^{x} f$. Median and quartiles solve $F = \frac{1}{2}, \frac{1}{4}, \frac{3}{4}$; the mode maximises f (calculus or corners). $E(X) = \int xf$, $E(X^2) = \int x^2 f$, $\text{Var} = E(X^2) - [E(X)]^2$. Skewness: mean < median < mode is negative skew; the reverse is positive.

Worked example – a quartic CDF (WST02 June 2025 Q1). $F(x) = k(6x^2 - x^4)$ on [0, 1]: $F(1) = 1$ gives $k = \frac{1}{5}$; $P(X > 0.6) = 1 - F(0.6) = 0.594$; the median solves $2m^4 - 12m^2 + 5 = 0$ with $m = 0.671$; $f = \frac{12x - 4x^3}{5}$ and $f' = 0$ give the mode 1; mean 0.64 < median < mode means negative skew.

2 Practice

2.1 $f(x) = kx^2$ on [0, 2]. Find k, $E(X)$ and $\text{Var}(X)$. [4]

2.2 For the same variable, find the median. [2]

2.3 $f(x) = \frac{2x}{9}$ on $[0, 3]$. Find $P(X > 1)$ and $E(X)$. [3]

3 Exam-style questions

3.1 *(Edexcel IAL WST02 June 2025 Q1, condensed.)* The continuous random variable X has cumulative distribution function $F(x) = k(6x^2 - x^4)$ for $0 \le x \le 1$ (0 below, 1 above). Given also that $E(X) = 0.64$: (a) show that $k = \frac{1}{5}$; (b) find $P(X > 0.6)$; (c) show that the median m satisfies $2m^4 - 12m^2 + 5 = 0$ and hence find m; (d) find, using calculus, the mode of X; (e) state, with a reason, the skewness of X. [11]

3.2 *(Original.)* $f(x) = kx^2(1 - x)$ on $[0, 1]$. (a) Show that $k = 12$. (b) Find $E(X)$ and $\text{Var}(X)$. (c) Find the mode of X. [6]