# 6.5 Discrete random variables

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

**KEY WORDS** discrete random variable • expectation

## Discrete random variables

## Objective

Use probability functions and cumulative functions of discrete random variables, compute E(X), E($X^2$), Var(X), apply linear coding, and quote the discrete uniform results (objectives 5.1–5.4).

## 1 Key knowledge and worked examples

**Routine.** Probabilities sum to 1 (find missing values). $F(x_0) = P(X \leq x_0)$. Solve inequalities for X BEFORE summing probabilities. $E(X) = \sum xp(x)$, $E(X^2) = \sum x^2 p(x)$, $\text{Var}(X) = E(X^2) - [E(X)]^2$; $E(aX + b) = aE(X) + b$, $\text{Var}(aX + b) = a^2\text{Var}(X)$. Uniform on $1..n$: $E(X) = \frac{n+1}{2}$, $\text{Var}(X) = \frac{n^2-1}{12}$.

**Worked example – the full routine (WST01 June 2025 Q1).** X on 1–4 with probabilities 0.3, 0.2, 0.35, 0.15: $F(3.5) = 0.85$; $3X - 3 < X + 2 \Rightarrow X < 2.5 \Rightarrow P = 0.5$; $E(X) = 2.35$; $E(X^2) = 6.65$; $\text{Var}(X) = 1.1275$; $\text{Var}(5 - 2X) = 4(1.1275) = 4.51$.

## 2 Practice

**2.1** X has $P(X = x) = \frac{kx}{6}$ for $x = 1, 2, 3$. Find $k$, $E(X)$ and $\text{Var}(X)$. [4]

**2.2** A fair die is rolled once. Using the uniform results, write down $E(X)$ and $\text{Var}(X)$. [2]

**2.3** Y has $E(Y) = 4$ and $\text{Var}(Y) = 2.5$. Find $E(3Y - 2)$ and $\text{Var}(3Y - 2)$. [2]

## 3 Exam-style questions

**3.1** *(Edexcel IAL WST01 June 2025 Q1.)* The discrete random variable X has probability distribution

| $x$ | **1** | **2** | **3** | **4** |
|---|---|---|---|---|
| $P(X = x)$ | 0.3 | 0.2 | 0.35 | 0.15 |

(a) Find $F(3.5)$. (b) Find $P(3X - 3 < X + 2)$. (c) Find $E(X)$. (d) Show that $\text{Var}(X) = 1.1275$. (e) Find $\text{Var}(5 - 2X)$. [9]

**3.2** *(Original.)* A game scores $X$ where $P(X = 0) = 0.25$, $P(X = 1) = 0.5$ and $P(X = 4) = 0.25$. The winnings are $W = 10X - 3$. (a) Find $E(W)$ and $\text{Var}(W)$. (b) Find $P(W > E(W))$. [6]