5.1 Vectors in mechanics

Name: _____ Class: _____ Date: _____ **Total: 25 marks**

KEY WORDS bearing • resultant

Vectors in mechanics

Objective

Work with displacements, velocities and forces as vectors in i, j form: magnitudes, resultants, position vectors at time t, relative position, and bearings (objectives 2.1–2.2).

1 Key knowledge and worked examples

Toolkit. $|a\mathbf{i} + b\mathbf{j}| = \sqrt{a^2 + b^2}$. Resultants add component-wise. Position at time t: $\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}$ for constant velocity, built from any known instant. Bearings are clockwise from north: with $\mathbf{i}$ east and $\mathbf{j}$ north, the vector $x\mathbf{i} + y\mathbf{j}$ with $x < 0$, $y < 0$ points south-west, at $\tan^{-1}(|x/y|)$ west of due south.

Worked example – position vectors and a bearing (WME01 June 2025 Q2). A has $\mathbf{r}_A(2) = -5\mathbf{i} + 16\mathbf{j}$, $\mathbf{r}_A(5) = 10\mathbf{i} + 4\mathbf{j}$: velocity $= \frac{15\mathbf{i}-12\mathbf{j}}{3} = 5\mathbf{i} - 4\mathbf{j}$, and $\mathbf{r}_A(t) = (5t-15)\mathbf{i} + (24-4t)\mathbf{j}$. B sits at $-5\mathbf{i} + 7\mathbf{j} + t(2\mathbf{i} - 3\mathbf{j})$; at $t = 5$, $\overrightarrow{AB} = -5\mathbf{i} - 12\mathbf{j}$, which is 22.6° west of south – bearing 203°.

2 Practice

2.1 Find the magnitude of the velocity $3\mathbf{i} - 4\mathbf{j}$ m s^{-1} and the bearing on which the particle moves. [3]

2.2 Forces $(2\mathbf{i} + \mathbf{j})$ N and $(4\mathbf{i} - 3\mathbf{j})$ N act on a particle. Find the magnitude and direction of the resultant force. [3]

2.3 A particle has position vector $3\mathbf{i} - 2\mathbf{j}$ m when $t = 1$ and $12\mathbf{i} + 7\mathbf{j}$ m when $t = 4$. Find its velocity and its position vector at time t seconds. [4]

2.4 Find the value of λ for which $\lambda\mathbf{i} + 2\mathbf{j}$ is perpendicular to $3\mathbf{i} - 4\mathbf{j}$. [3]

3 Exam-style questions

3.1 *(Edexcel IAL WME01 June 2025 Q2.)* At time t seconds, the position vector of particle A is $\mathbf{r}$ metres. When $t = 2$, $\mathbf{r} = (-5\mathbf{i} + 16\mathbf{j})$; when $t = 5$, $\mathbf{r} = (10\mathbf{i} + 4\mathbf{j})$. Particle B has position vector $\mathbf{s} = -5\mathbf{i} + 7\mathbf{j} + t(2\mathbf{i} - 3\mathbf{j})$ metres. (a) Find the velocity of A. (b) Find $\mathbf{r}$ in terms of t. (c) Find the bearing of B from A when $t = 5$, giving your answer to the nearest degree. [7]

3.2 *(Original.)* A ship leaves the point with position vector $(2\mathbf{i} - \mathbf{j})$ km and moves with constant velocity $(3\mathbf{i} + 4\mathbf{j})$ $\mathrm{km\,h^{-1}}$, where $\mathbf{i}$ and $\mathbf{j}$ point east and north. (a) Find the bearing of the ship from the origin at the moment it leaves. (b) Find t when the ship is due east of the origin, and its distance from the origin then. [5]